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OSCILLATING SYSTEMS WITH ONE VARIABLE ELEMENT

BY

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ABSTRACT

A passive electrical circuit composed of capacitance, inductance, and resistance in series is analyzed with the condition that the capacitance is caused to change monotonically with time. It is further assumed that dissipative effects are small enough to allow oscillation in the circuit. The changing capacitance produces changes in both the instantaneous frequency and the amplitude of oscillation. Amplitude changes can be related to the energy required to produce the change in capacitance. By causing the capacitance to vary appropriately, the amplitude of oscillation can be maintained constant, even though dissipative effects tend to make it decay. The instantaneous frequency may change considerably, however, under these conditions.

Analogous mechanical systems are also considered briefly.

I. INTRODUCTION

A controlled variation of one element of a passive oscillatory system may produce interesting modifications in the usual mode of oscillation. Changes in value of an energy-storing element must cause corresponding changes in the circulating energy. The case of periodic variations in a system with a single degree of freedom has been investigated in considerable detail, both analytically and experimentally (1, 2, 3, 4).[2] The simplest case is that where one energy-storing element is made to vary as a simple harmonic function of time, so that the system is described by a Mathieu equation. The nature of the resulting oscillation depends upon the ratio between the natural frequency of the system and the frequency at which the element varies, as well as upon the relative magnitude of the variation. The system may oscillate with periodically varying amplitude and frequency, in which case it is said to be stable.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

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Alternatively, it may oscillate with amplitude increasing indefinitely, in which case it is said to be unstable. If sufficient dissipation is present in the latter case, the effect is to counterbalance the tendency for the amplitude to increase, and instability does not actually occur. Electrical systems having a reactive element caused to vary periodically with time are presently being investigated under the name of parametric amplifiers (5). They offer considerable promise as a means of obtaining power amplification at high frequencies with little accompanying noise.

The energy-storing element can also be caused to vary in a manner other than periodic. Again, energy interchanges take place and characteristics of the oscillation change. Element variations that are either linear or exponential functions of time are relatively simple to study and lead to phenomena that are worth investigating. Physical systems of the type considered may be either electrical with capacitance and inductance, or mechanical with mass and stiffness.

II. SERIES ELECTRICAL CIRCUIT

An electrical circuit with inductance L, resistance R, and capacitance C, connected in series, is considered first. It is governed by the equation

$$(d/dt)(L\dot{q}) + R\dot{q} + q/C = 0 \tag{1}$$

where t is time, dots indicate differentiation with respect to time, and q is the charge upon the capacitor. The capacitance is assumed to change with time as

$$C(t) = C_0 f(t) \tag{2}$$

where C_0 is a constant and $f(t)$ is the time function describing the variation. A combination of Eqs. 1 and 2 gives

$$\ddot{q} + 2P\dot{q} + \omega_0^2 f^{-1}(t)q = 0 \tag{3}$$

where $P = R/2L$, $\omega_0^2 = 1/LC_0$, and both R and L are assumed to be constant. This is a linear second-order equation with a varying coefficient. An approximate solution for it can be found by the WKBJ method (6, 7). In order to apply this method, the term with the first derivative must be removed, which can be done through a change of variable from q to y, where $y = q \exp(\int P dt)$. The resulting equation is

$$\ddot{y} + G^2(t)y = 0 \tag{4}$$

where

$$G^2(t) = \omega_0^2 f^{-1}(t) - P^2 - \dot{P}. \tag{5}$$

In the present case where $P = R/2L$ and P is constant, $\dot{P}$ is zero, of course. By the WKBJ method, an approximate solution for Eq. 4 is

known to be

$$y = G^{-1/2}(t)M(t) \tag{6}$$

where $M(t) = A \cos \phi + B \sin \phi$ with A and B arbitrary constants, and $\phi = \phi(t) = \int G(t)dt$. This solution is of high accuracy so long as the following requirement is satisfied

$$|G^2| \gg |(\ddot{G}/2G) - (3/4)(\dot{G}/G)^2|. \tag{7}$$

In many cases of practical interest, where oscillations are but lightly damped, the dissipative effects are relatively small and terms in P in Eq. 5 can be neglected. The requirement of Eq. 7, when used with the type of variation of Eq. 2, then becomes

$$\omega_0^2 \gg |3\dot{f}^2/16f - \ddot{f}/4|. \tag{8}$$

This sets a limitation upon the relative rate of change of capacitance as compared with the natural frequency of the circuit. The charge in the circuit can be found by transferring the solution for variable y, in Eq. 6, back to variable q, and is

$$q = \exp(-\int P dt) G^{-1/2}(t) M(t). \tag{9}$$

A specific example is that in which the capacitance is made to vary linearly with time, so that

$$f(t) = (1 + t/\tau) \tag{10}$$

where τ is the interval required for the capacitance to change by an amount C_0. At the same time, quantity P is $P = R/2L$ and is constant and assumed to be much smaller than ω_0^2. The charge is, from Eq. 9,

$$q = \exp(-Pt)\omega_0^{-1/2} f^{1/4}(t) M(t). \tag{11}$$

The charge is oscillatory with amplitude which tends to decay as $\exp(-Pt)$ because of dissipative action, and tends to grow as $(1 + t/\tau)^{1/4}$ because of the term $f^{1/4}(t)$. The instantaneous angular frequency of the oscillating function, $M(t)$, is

$$\omega = d\phi/dt = G(t) = \omega_0(1 + t/\tau)^{-1/2}. \tag{12}$$

This frequency decreases with increasing time.

In this case of linearly varying capacitance, the restriction of Eq. 8 becomes $\omega_0^2\tau^2(1 + t/\tau) \gg 3/16$, and at $t = 0$ this reduces to $\omega_0^2\tau^2 \gg 3/16$. If T_0 is the period of oscillation, corresponding to ω_0 at $t = 0$, then $T_0 = 2\pi/\omega_0$ and the restriction becomes $\tau \gg (3/16)^{1/2}(T_0/2\pi) = 0.069T_0$. Thus, so long as the capacitance doubles, or halves, in a time at least

as long as the period of oscillation, the approximate solution should be relatively accurate.

An idealized case is that in which there is no dissipation, so that both $R = 0$ and $P = 0$. Then, there is no damping action and the amplitude of oscillation of the charge grows as $f^{1/4}(t)$, according to Eq. 11. The current in the circuit is the time derivative of the charge, so that the current i is

$$i = dq/dt = \omega_0^{1/2} f^{-1/4}(t) N(t) + \tfrac{1}{4}\omega_0^{-1/2} f^{-3/4}(t) \dot{f}(t) M(t) \qquad (13)$$

as found from Eq. 11 with $P = 0$. In Eq. 13, function $N(t)$ is defined as $N(t) = -A \sin \phi + B \cos \phi$. Provided $|\dot{f}| < |f^2/f|$, the restriction of Eq. 8 is such that the second term in Eq. 13 can be neglected. For the linear capacitance change, $\dot{f}$ is zero, of course. Very nearly, then, the current is

$$i = \omega_0^{1/2} f^{-1/4}(t) N(t). \qquad (14)$$

The current amplitude decreases as $f^{-1/4}(t)$. The voltage e across the capacitance is given by

$$e = q/C = C_0^{-1} \omega_0^{-1/2} f^{-3/4}(t) M(t) \qquad (15)$$

as found from Eqs. 2 and 11. The flux linkage in the inductance is given by

$$\Phi = Li = L\omega_0^{1/2} f^{-1/4}(t) N(t) \qquad (16)$$

as found from Eq. 14.

III. ENERGY INTERCHANGES

All these quantities, charge, current, voltage, and flux, are oscillatory with amplitudes which change because of the change in capacitance. The change in amplitude represents a change in circulating energy in the circuit. This energy change must be supplied by the mechanism providing the change in capacitance. In order to investigate this interaction, the capacitance can be imagined as made up of two parallel plates, with the capacitance being increased by reducing the spacing between the plates. If a voltage exists across the plates, there is a force of attraction F between the plates given by the relation

$$F = -\tfrac{1}{2} e^2 dC/dx \qquad (17)$$

where x is the separation of the plates. The negative sign is required because an increase in x reduces C, and positive F is directed oppositely to positive x. If the separation is caused to change from x_1 to x_2, the mechanical work W which must be done by the outside agency pro-

ducing the change is

$$W = \int_{x_1}^{x_2} F dx = -\frac{1}{2}\int_{x_1}^{x_2} e^2 (dC/dx) dx$$

$$= -\frac{1}{2}\int_{C_1}^{C_2} e^2 dC \tag{18}$$

where C_1 and C_2 are initial and final values of capacitance. In a dissipationless circuit, the voltage between the plates is given by Eq. 15. If, for simplicity, conditions are chosen so that coefficient B in $M(t)$ is zero, then

$$e^2 = C_0^{-2}\omega_0^{-1} f^{-3/2}(t) A^2 \cos^2\phi(t).$$

Furthermore, since $C = C_0 f(t)$, then $f^{-3/2}(t) = (C/C_0)^{-3/2}$. When this value of e^2 is used in Eq. 18, the resulting integral is generally too complicated to evaluate. However, if the interval during which the capacitance changes is taken as an integral number of half cycles of the cosine function, the average of the cosine squared can be used with small error. This average is one-half. Thus, the work done becomes

$$W = -\frac{1}{2}\int_{C_1}^{C_2} C_0^{-1/2}\omega_0^{-1} C^{-3/2}(A^2/2) dC$$

$$= (A^2/2\omega_0 C_0^{1/2})(1/C_2^{1/2} - 1/C_1^{1/2}). \tag{19}$$

Since C is assumed to increase with time, C_2 exceeds C_1, and W is a negative quantity. Therefore, work is done on the outside agency controlling the variation in capacitance.

Some justification for the use of the average of the cosine-squared function in obtaining Eq. 19 can be offered as follows. The analysis using the WKBJ method, and leading to Eq. 15 for voltage e, is based on the condition that the capacitance varies only relatively slowly. The restriction of Eq. 8 requires that the relative capacitance change, per cycle of oscillation, be not too large. Thus, the relation for e^2 represents a cosine-squared function with amplitude varying slowly. This cosine-squared function does have the property that its angle $\phi(t)$ increases as a nonlinear function of time t. In fact, since quantity $P = 0$, as previously assumed, the angle is given as a function of time by the relation

$$\phi(t) = \int G dt = \int \omega_0 f^{-1/2} dt = \omega_0 C_0^{-1/2} \int f^{-1} C^{-1/2} dC.$$

For the particular case of linear variation of capacitance as given by Eq. 10, $f = 1/\tau$ and the angle becomes $\phi(t) = 2\omega_0 \tau C_0^{-1/2} C^{1/2}$, varying as $C^{1/2}$. Finding the average between successive zeros of $\cos^2\phi$, where

ϕ has the relation above, is equivalent to evaluating the integral

$$(2/\pi^2)\int_{-\pi^2/4}^{+\pi^2/4}\cos^2(\theta^{1/2})d\theta,$$

which is, indeed, one-half. Therefore, even though the cosine function has decreasing instantaneous frequency, the average of its square over half-cycle intervals is still one-half, and this value can be used in obtaining the integral of Eq. 19.

The work represented by Eq. 19 is related to the change in circulating energy of the circuit. At the instant when the voltage across the capacitance is a maximum, e_m, all the circulating energy is stored in the capacitance and is

$$W_m = \tfrac{1}{2}Ce_m^2 = \tfrac{1}{2}CC_0^{-2}\omega_0^{-1}(C/C_0)^{-3/2}A^2.$$

The difference between this maximum stored energy for the two values of capacitance, C_1 and C_2, is

$$W_{m2} - W_{m1} = (A^2/2\omega_0 C_0^{1/2})(1/C_2^{1/2} - 1/C_1^{1/2}). \qquad (20)$$

This difference is the same as the work of Eq. 19. Thus, as would be expected, the reduction in circulating energy of the system appears as work done upon the outside agency controlling the changing capacitance.

The foregoing discussion applies to a linear change in capacitance with the fractional change per cycle of oscillation being not too large. The voltage amplitude then varies as $f^{-3/4}(t)$, or $C^{-3/4}$, from Eq. 15. If the capacitance is caused to change more rapidly, the preceding analysis is no longer valid. However, an alternate line of reasoning can be used to obtain some information about the limiting case in which the capacitance jumps suddenly from one value to another in essentially zero time. In this case the resulting effect depends upon the instant in the cycle of oscillation at which the jump occurs. If the jump takes place at the instant when the voltage across the capacitance is zero, no change in energy is involved and the quantity $\frac{1}{2}e_m^2C$ is a constant. The voltage amplitude is then proportional to $C^{-1/2}$. Alternatively, if the jump takes place at the instant of maximum voltage across the capacitance, the charge must remain constant. This is evident, since zero time is allowed, and any change in charge during this time would require infinite current, which is prevented by the series inductance. Since the voltage is, in general, $e = q/C$, in this case where q is constant the voltage amplitude is proportional to C^{-1}. Thus, the voltage amplitude, which varies as $C^{-3/4}$ where relatively slow changes in C take place, may vary as $C^{-1/2}$ or C^{-1} when sudden jumps in C occur.

Analysis of the dissipationless electrical circuit, with linearly in-

creasing capacitance, has shown that the charge increases indefinitely with time, as given by Eq. 11. The question might arise as to whether this case would be considered as unstable. Other quantities, current, voltage, and flux, given in Eqs. 14, 15 and 16, decrease as the capacitance increases with time. Energy in the system decreases, as shown by Eq. 20. For this reason, it would appear that the circuit with increasing capacitance should be classified as stable.

On the other hand, if the capacitance is caused to decrease linearly with time, energy in the system does increase. This case might, therefore, be termed unstable. Actually, as the capacitance decreases linearly at a constant rate, the instantaneous frequency of oscillation continually increases. Eventually the relative decrease in capacitance per cycle of oscillation is no longer small, and the entire analysis becomes invalid. Furthermore, the additional energy appearing in the circuit must be supplied by the mechanism causing the capacitance to decrease. In any physical system, only a finite amount of energy can be supplied, and the energy in the system can never actually become infinite.

IV. EXPERIMENTAL TESTS

The result of an experimental investigation of the circuit with zero resistance and linear variation of capacitance is shown in Fig. 1. These curves were obtained using an analog computer. Both charge q and current $i = dq/dt$ are plotted. Two rates of change of capacitance, as described by parameter τ, are shown. The larger value, τ_2, is five times the smaller, τ_1, and represents a slower variation of capacitance. In Eq. 11 it is shown that the amplitude of q varies as $f^{1/4}(t) = \tau^{-1/4}(t + \tau)^{1/4}$. For this reason, the abscissa of Fig. 1 is chosen as $(t + \tau)$. In each case, the initial conditions at $t = 0$ are chosen as $i = 0$ and q a constant, with this constant proportional to $\tau^{1/4}$. It is evident from Fig. 1 that both q and i are oscillatory, with the intervals between successive zeros increasing with time, as required by the change in instantaneous frequency given by Eq. 12. At the same time, the amplitude of charge q grows as $(t + \tau)^{1/4}$ while the amplitude of current i decreases as $(t + \tau)^{-1/4}$, as required by Eqs. 11 and 14.

When resistance is present in the circuit, the amplitude of oscillation of the charge tends to decay as $\exp(-Pt)$, as given by Eq. 11. Evidently, by choosing properly the way in which the capacitance varies with time, the amplitude of the charge can be kept constant. The necessary variation is given by $f(t) = \exp(4Pt)$, also from Eq. 11. An experimental test of this kind of variation is shown in Fig. 2. The upper curve of this figure represents the charge in a circuit with resistance, decaying exponentially in amplitude as it oscillates. The circuit Q in this example is $Q = 16.7$. Successive zeros of charge are, of course, equally spaced. The central curve of this figure represents the

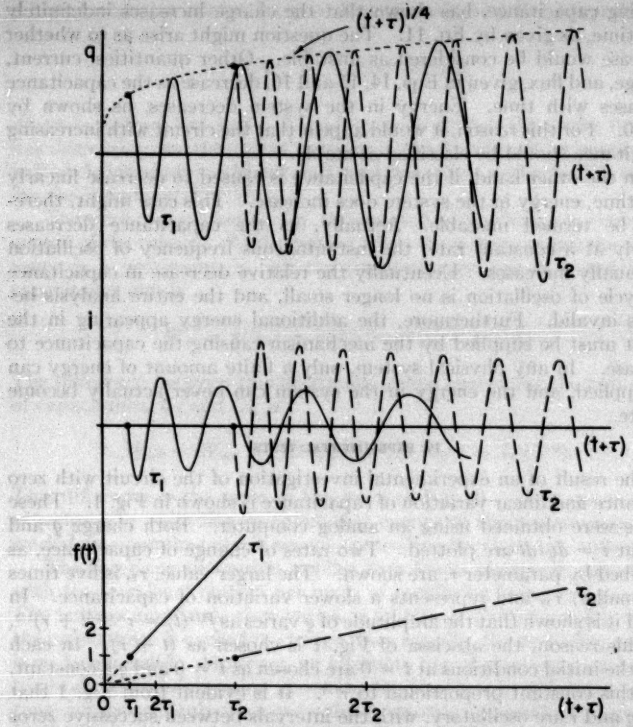

FIG. 1. Curves obtained with an analog computer showing charge q and current i as a function of time t in a dissipationless electrical circuit. The circuit consists of a constant inductance and a capacitance varying linearly with time, as $f(t) = (1 + t/\tau)$. Two values of τ are used, with $\tau_2 = 5\tau_1$. As C increases, both q and i oscillate with decreasing angular frequency. The amplitude of q grows while that of i decays.

charge in the same circuit, except now the capacitance is caused to increase exponentially with time. The capacitance variation is shown in the lower curve. The charge of the central curve oscillates with constant amplitude, but the zeros are spaced more and more widely as the capacitance increases with time. It is apparent that a large variation in capacitance is needed to compensate for the decay due to even a relatively small amount of dissipation.

Just as it is possible to compensate for the amplitude decay produced by dissipative effects in the case of charge, so also can compensation be

achieved for current, voltage, or flux. The necessary capacitance variation for each of these cases can be found from Eqs. 14, 15 and 16.

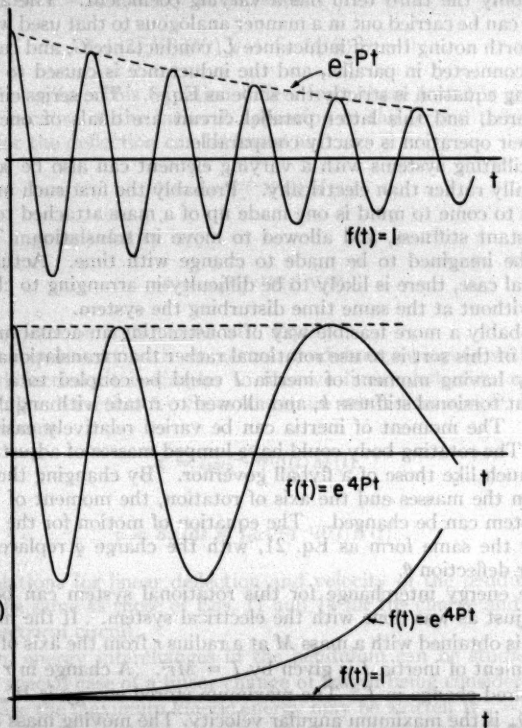

FIG. 2. Curves obtained with an analog computer showing charge q as a function of time t in an electrical circuit with dissipation. The capacitance is constant in one case, $f(t) = 1$, and in the second case varies as $f(t) = \exp(4Pt)$. The varying capacitance compensates for the decay in amplitude of oscillation produced by dissipative effects. At the same time, the instantaneous frequency decreases considerably.

V. OTHER EXAMPLES OF OSCILLATORY SYSTEMS

While all the foregoing discussion applies to an electrical circuit with varying capacitance, much the same kind of effects occur if the other energy-storing element, the inductance, is made to vary. The governing equation is, from Eq. 1,

$$\ddot{q} + [(\dot{L} + R)/L]\dot{q} + q/LC = 0. \tag{21}$$

This differs from Eq. 3, for capacitance variation, in that coefficients of both the second and third terms are now functions of time, while in Eq. 3 only the third term has a varying coefficient. The analysis of Eq. 21 can be carried out in a manner analogous to that used with Eq. 3. It is worth noting that if inductance L, conductance G, and capacitance C are connected in parallel, and the inductance is caused to vary, the resulting equation is strictly the same as Eq. 3. The series circuit, first considered, and this latter parallel circuit are duals of one another, and their operation is exactly comparable.

Oscillating systems with a varying element can also be set up mechanically rather than electrically. Probably the first such mechanical system to come to mind is one made up of a mass attached to a spring of constant stiffness, and allowed to move in translation. The mass could be imagined to be made to change with time. Actually, in a practical case, there is likely to be difficulty in arranging to change the mass without at the same time disturbing the system.

Probably a more feasible way of constructing an actual mechanical system of this sort is to use rotational rather than translational motion. A body having moment of inertia I could be coupled to a spring of constant torsional stiffness k, and allowed to rotate with angular deflection θ. The moment of inertia can be varied relatively easily in this case. The rotating body could have lumped masses of adjustable position, much like those of a flyball governor. By changing the distance between the masses and the axis of rotation, the moment of inertia of the system can be changed. The equation of motion for the system is exactly the same form as Eq. 21, with the charge q replaced by the angular deflection θ.

The energy interchange for this rotational system can be investigated, just as was done with the electrical system. If the moment of inertia is obtained with a mass M at a radius r from the axis of rotation, the moment of inertia I is given by $I = Mr^2$. A change in r produces the desired change in I. The maximum stored energy is $W_m = \frac{1}{2}I\dot{\theta}_m^2$, where $\dot{\theta}_m$ is the maximum angular velocity. The moving mass is subject to a centrifugal force $F = Mr\dot{\theta}^2$, and the work done on the mass as it moves outward to increase the moment of inertia is $W = \int_{r_1}^{r_2} Mr\dot{\theta}^2 dr$. It is a simple matter to show that this work is just equal to the difference in maximum stored energy for the two values of radius r, using the same argument as was used with the electrical circuit.

Another mechanical system that is easy to visualize is a simple pendulum in which the length l is made to vary with time as $l = l_0 f(t)$. The equation of motion of the pendulum is

$$Ml^2\ddot{\theta} + 2Ml\dot{l}\dot{\theta} + Mgl \sin\theta = 0 \qquad (22)$$

where M is the mass concentrated at length l, the angular deflection is θ, and g is the gravitational acceleration. If deflections are small enough, $\theta \ll 1$ and $\sin\theta = \theta$, and Eq. 22 becomes

$$\ddot{\theta} + 2[\dot{f}(t)/f(t)]\dot{\theta} + [\omega_0^2/f(t)]\theta = 0 \tag{23}$$

where $\omega_0^2 = g/l_0$. This equation differs from those found in previous examples because of the factor 2 appearing in the second term. The solution for the deflection can be found as

$$\theta = \omega_0^{-1/2} f^{-3/4}(t) M(t) \tag{24}$$

while the angular velocity is

$$\dot{\theta} = \omega_0^{1/2} f^{-5/4}(t) N(t) \tag{25}$$

where the notation is that used with Eq. 13.

In an experiment with an actual pendulum, swinging with small angular deflection, it is simpler to observe linear deflection x, rather than angular deflection θ. Since these are related as $x = l\theta$, Eqs. 24 and 25 become

$$x = l_0 \omega_0^{-1/2} f^{1/4}(t) M(t) \tag{26}$$

and

$$v = dx/dt = l_0 \omega_0^{1/2} f^{-1/4}(t) N(t). \tag{27}$$

These relations for linear deflection and velocity of the pendulum are exactly the same as those of Eqs. 11 and 14 for the charge and current in the electrical circuit.

Again, energy interchanges in the pendulum can be studied, with only the special case of a linear change of length being considered here. In this case the maximum kinetic energy may be written $W_m = \frac{1}{2} M l^2 \dot{\theta}_m^2 + \frac{1}{2} M \dot{l}^2$. If the pendulum is thought of as a mass on the end of a string, the tension in the string is $F = M l \dot{\theta}^2 + Mg \cos\theta$. The first term is the centrifugal force and the second term is the result of gravitational attraction of the mass. Under the assumption of small deflection, $\cos\theta = 1 - \theta^2/2$ and $Mg\cos\theta = Mg - Mg\theta^2/2$. The first term here is the steady gravitational force, and can be ignored so far as changes in kinetic energy are concerned. The work done on the mass as the length of the pendulum increases is then $W = \int_{l_1}^{l_2} (M l \dot{\theta}^2 - Mg\theta^2/2) dl$. It is once more a simple matter to show that this work is just the change in maximum kinetic energy as the length changes.

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A NOTE ON THE MANLEY-ROWE ENERGY RELATIONS FOR PARAMETRIC AMPLIFIERS

BY

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1. INTRODUCTION

Manley and Rowe have proved (1)[2] two relations governing the flow of energy in circuits containing non-linear reactances. These important relations form the basis of the theory of parametric amplifiers and of other related devices (2). Manley and Rowe's analysis is complete, ingenious, and elegant. The purpose of this note simply is to give an alternative elementary derivation that clearly exhibits the intuitive "meaning" of the relations (3).[3]

The circuit analyzed is shown in Fig. 1. It consists of a non-linear reactor in parallel with several branches. One branch contains a load,

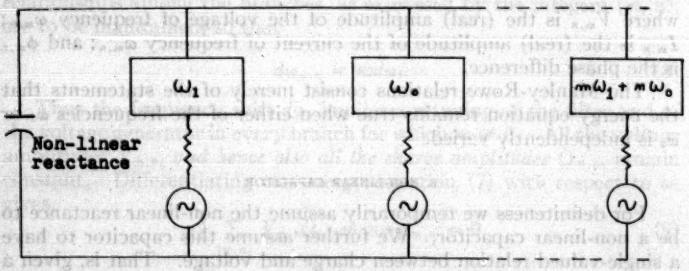

FIG. 1.

a voltage generator of frequency ω_1, and an ideal filter that passes only the frequency ω_1. A second branch has similar structure, with frequency ω_0. Other branches repeat this structure, with frequencies $\omega_{m,n}$, given by

$$\omega_{m,n} = m\omega_1 + n\omega_0. \tag{1}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] The author is indebted to Dr. James P. Wittke, of the R.C.A. Laboratories at Princeton, who has pointed out to me the existence of two other simplified derivations of the Manley-Rowe relations. These are given in reference 3. That of Salzberg is closely related to the derivation in this paper, although the physical point of view adopted is somewhat different.

The coefficients m and n are integers (positive, negative, or zero) so chosen that $\omega_{m,n}$ is always positive. This convention is somewhat different than that adopted by Manley and Rowe; in this note all frequencies are positive and all currents and voltages are real.

Let $W_{m,n}$ be the real, average power flowing into the non-linear capacitor, with frequency $\omega_{m,n}$. Then $W_{m,n}$ is also the real, average power flowing out of the (m, n) branch; it equals the power supplied by the (m, n) generator diminished by the power dissipated in the (m, n) load. The role of the non-linear reactor is merely that of a frequency shifter for the various power components, and it does not act as either a source or sink of energy. Hence the conservation of energy requires that

$$\sum_{(m,n)} W_{m,n} = 0. \tag{2}$$

It is convenient to rewrite this equation in the form

$$\tfrac{1}{2} \sum_{(m,n)} V_{m,n} I_{m,n} \cos \phi_{m,n} = 0, \tag{3}$$

where $V_{m,n}$ is the (real) amplitude of the voltage of frequency $\omega_{m,n}$; $I_{m,n}$ is the (real) amplitude of the current of frequency $\omega_{m,n}$; and $\phi_{m,n}$ is the phase difference.

The Manley-Rowe relations consist merely of the statements that the energy equation remains true when either of the frequencies ω_1 or ω_0 is independently varied.

2. NON-LINEAR CAPACITOR

For definiteness we temporarily assume the non-linear reactance to be a non-linear capacitor. We further assume this capacitor to have a single-valued relation between charge and voltage. That is, given a particular set of voltage amplitudes $V_{m,n}$, the capacitor generates a definite set of charge amplitudes $Q_{m',n'}$. *The transformation from the set of voltages* $V_{m,n}$ *to the set of charges* $Q_{m',n'}$ *depends only on the integers* m, n, m′, n′ *and not on the values of the frequencies* ω_0 *and* ω_1.

As a specific example, suppose that the capacitor satisfies the relation

$$Q = CV^2. \tag{4}$$

If we apply the particular voltage

$$V = V_{0,1} \cos \omega_{0,1} t + V_{1,0} \cos \omega_{1,0} t \tag{5}$$

the charge is easily computed. In fact, given the voltage components $V_{0,1}$ and $V_{1,0}$ the capacitor generates charge components in accordance

with the transformation:

$$\left\{\begin{matrix} V_{0,1} \\ V_{1,0} \end{matrix}\right\} \rightarrow \left\{\begin{matrix} Q_{0,0} = \frac{1}{2}CV_{0,1}^2 + \frac{1}{2}CV_{1,0}^2 \\ Q_{0,2} = \frac{1}{2}CV_{0,1}^2 \\ Q_{2,0} = \frac{1}{2}CV_{1,0}^2 \\ Q_{1,1} = CV_{0,1}V_{1,0} \\ Q_{-1,1} = CV_{0,1}V_{1,0} \end{matrix}\right\}. \tag{6}$$

The important feature of this transformation is that it does *not* depend on the frequencies $\omega_{0,1}$ and $\omega_{1,0}$; it remains true no matter what values these frequencies or their ratio may assume.

By contrast the transformation from the voltage amplitudes $V_{m,n}$ to the current amplitudes $I_{m,n} = \omega_{m,n}Q_{m,n}$ does depend on the frequencies.

Because the $Q_{m,n}$ are more fundamentally related to the voltage amplitudes than are the $I_{m,n}$, we rewrite the energy equation as

$$\tfrac{1}{2} \sum_{(m,n)} V_{m,n}Q_{m,n}\omega_{m,n} \cos \phi_{m,n} = 0. \tag{7}$$

We now consider a small change $d\omega_1$ in the frequency ω_1. All interrelationships among the branches, as expressed by the integers (m, n), are to be maintained, so that

$$d\omega_{m,n} = m d\omega_1. \tag{8}$$

Thus the frequency shift $d\omega_1$ implies a retuning of the filter and of the voltage generator in every branch for which $m \neq 0$. All the voltage amplitudes $V_{m,n}$, *and hence also all the charge amplitudes* $Q_{m,n}$, remain constant. Differentiating the energy equation (7) with respect to ω_1 gives

$$\tfrac{1}{2} \sum_{(m,n)} V_{m,n}Q_{m,n}m \cos \phi_{m,n} = 0. \tag{9}$$

Replacing the charge amplitudes $Q_{m,n}$ by $\dfrac{I_{m,n}}{\omega_{m,n}}$ gives directly

$$\sum_{(m,n)} \frac{mW_{m,n}}{\omega_{m,n}} = 0. \tag{11}$$

This is the first of the Manley-Rowe relations. The second relation is obtained by considering a variation in ω_0, and by direct analogy it is

$$\sum_{(m,n)} \frac{nW_{m,n}}{\omega_{m,n}} = 0. \tag{11}$$

The two Manley-Rowe relations are not independent of the energy relation (2); in fact multiplication of Eq. 10 by ω_0, of Eq. 11 by ω_1, and addition give Eq. 2.

3. NON-LINEAR INDUCTANCE

Assume the non-linear reactor to be an inductor, with a single-valued relation between the voltage V and the time rate of change of current, $\dot{I}$. Given a particular set of voltage amplitudes $V_{m,n}$, the inductor generates a definite set of amplitudes $\dot{I}_{m',n'}$. *The transformation from the set of voltage amplitudes* $V_{m,n}$ *to the amplitudes* $\dot{I}_{m',n'}$ *depends only on the integers* m, n, m′, n′ *and not on the values of the frequencies* ω_0 *and* ω_1.

In the energy equation (3) we replace the current amplitude $I_{m,n}$ by $\dot{I}_{m,n}/\omega_{m,n}$, and differentiate with respect to ω_1.

$$-\tfrac{1}{2}\sum_{(m,n)} V_{m,n}\frac{\dot{I}_{m,n}}{\omega_{m,n}^2}\, m\cos\phi_{m,n} = 0. \tag{13}$$

Replacing the $\dot{I}_{m,n}/\omega_{m,n}$ by $I_{m,n}$, we again obtain the Manley-Rowe relation (10).

The usefulness of the Manley-Rowe relations has been discussed extensively elsewhere. We have merely shown that these relations can be understood in an elementary way, as the requirements of energy conservation under variations of the fundamental frequencies of the given circuit.

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SECOND-ORDER NON-LINEAR DIFFERENCE EQUATIONS CONTAINING SMALL PARAMETERS

BY

H. C. TORNG [1]

ABSTRACT

This paper presents a method for solving second-order non-linear difference equations containing small parameters. The theory of first approximation, which is widely used in solving non-linear differential equations (1, 2),[2] is extended to the field of non-linear difference equations. This approach furnishes either exact or approximate solutions and reveals any possible self-sustained recurrent oscillations.

1. INTRODUCTION

In the study of pulsed-servomechanisms, magnetic amplifiers, quality control and other subjects, the investigators often are confronted with a set of difference equations. Linear systems can be taken care of by linear difference equations with variable or constant coefficients. The presence of non-linearities in a physical system brings about non-linear difference equations.

Difference equations can also be considered as recurrent relations. With appropriate given initial conditions, the solution is uniquely determined and can be computed step by step; but, by so doing, the correlation between the system performance and system parameters is entirely missed.

In studying a physical system, the investigation of possible self-oscillations is of prime interest. To carry out the step-by-step calculation for all possible initial situations is prohibitively cumbersome and will be of little value to synthesis.

This paper deals specifically with second-order non-linear difference equations containing small parameters. In considering that such a non-linear equation is a deviation from a linear one, the latter is first studied in the next section.

2. A SET OF LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

The following set of equations is first investigated.

$$\begin{aligned} X_{n+1} &= A_{11}X_n + A_{12}Y_n \\ Y_{n+1} &= A_{21}X_n + A_{22}Y_n \end{aligned} \tag{1}$$

where A_{11}, A_{12}, A_{21} and A_{22} are constants, X_n and Y_n are the dependent

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

variables of the equations, and n is the independent variable, taking integral values.

Equation 1 implies the following equations

$$\begin{aligned} X_{n+2} - (A_{22} + A_{11})X_{n+1} + (A_{11}A_{22} - A_{12}A_{21})X_n &= 0 \\ Y_{n+2} - (A_{11} + A_{22})Y_{n+1} + (A_{11}A_{22} - A_{12}A_{21})Y_n &= 0. \end{aligned} \tag{2}$$

Let

$$\begin{aligned} -(A_{11} + A_{22}) &= A_1 \\ A_{11}A_{22} - A_{12}A_{21} &= A_2. \end{aligned} \tag{3}$$

Equation 2 becomes

$$X_{n+2} + A_1X_{n+1} + A_2X_n = 0. \tag{4}$$

The solution of Eq. 4 can be expressed as

$$X_n = C_1\beta_1^n + C_2\beta_2^n, \tag{5}$$

where

$$\beta_{1,2} = \tfrac{1}{2}[-A_1 \pm (A_1^2 - 4A_2)^{1/2}]. \tag{6}$$

C_1 and C_2 can be considered either as constants or functions of period 1.

From Eq. 6, it is clear that oscillations can occur only when

$$4A_2 \geqslant A_1^2, \tag{7}$$

that is

$$4(A_{11}A_{22} - A_{12}A_{21}) \geqslant (A_{11} + A_{22})^2.$$

When Eq. 7 is fulfilled, β_1 and β_2 can be written as follows

$$\begin{aligned} \beta_1 &= -\frac{A_1}{2} + \tfrac{1}{2}(4A_2 - A_1^2)^{1/2}i = \rho e^{i\theta} \\ \beta_2 &= -\frac{A_1}{2} - \tfrac{1}{2}(4A_2 - A_1^2)^{1/2}i = \rho e^{-i\theta} \end{aligned} \tag{8}$$

where

$$\rho^2 = \left(-\frac{A_1}{2}\right)^2 + \tfrac{1}{4}(4A_2 - A_1^2) = A_2 \tag{9}$$

$$\theta = \tan^{-1}\frac{(4A_2 - A_1^2)^{1/2}}{-A_1}. \tag{10}$$

The solution of Eq. 4 is

$$\begin{aligned} X_n &= C_1(\rho e^{i\theta})^n + C_2(\rho e^{-i\theta})^n \\ &= \rho^n K \sin(n\theta + \phi) \end{aligned} \tag{11}$$

where K and ϕ take the place of C_1 and C_2 as constants.

To have a sustained oscillation, which is of prime interest in this study, ρ has to be unity, that is,

$$A_2 = A_{11}A_{22} - A_{12}A_{21} = 1. \tag{12}$$

When Eq. 11 is examined closely, it is obvious that this oscillation has a special recurrent property. This stems from the fact that n takes integral values. When $2\pi/\theta = C$ is an integer, the solution would trace the same C points during each period. If C is not an integer, the solution remains on a circle, but progresses "recurrently" (3, 4).

3. SECOND ORDER NON-LINEAR DIFFERENCE EQUATION CONTAINING SMALL PARAMETERS

The following equation

$$X_{n+2} + A_1X_{n+1} + A_2X_n = \mu f(X_n, X_{n+1}) \tag{13}$$

will be investigated.

Where μ is a small parameter, $f(X_n, X_{n+1})$ is a non-linear function of X_n and X_{n+1}, defined $0 \leqslant n < N$ and $N \to \infty$.

To solve Eq. 13, or try to approximate its exact solution, the right hand side of Eq. 13 is, at first, considered as a forcing function.

The solution of the homogeneous equation

$$X_{n+2} + A_1X_{n+1} + A_2X_n = 0 \tag{13a}$$

is

$$X_n = A(\rho e^{i\theta})^n + B(\rho e^{-i\theta})^n. \tag{14}$$

If A_1 and A_2 satisfy the conditions of Eqs. 7 and 12, and $\rho = 1$, then

$$X_n = Ae^{i\theta n} + Be^{-i\theta n}. \tag{15}$$

To take into account the presence of the terms on the right hand side of Eq. 13, A and B of Eq. 15 are considered as functions of the integral independent variable n instead of constants or periodic functions with period 1 as originally designated in Eq. 15, that is,

$$X_n = A(n)e^{i\theta n} + B(n)e^{-i\theta n}. \tag{16}$$

Since

$$\begin{aligned} A_{n+1} &= A_n + \Delta A_n \\ B_{n+1} &= B_n + \Delta B_n, \end{aligned} \tag{17}$$

from Eq. 16

$$\begin{aligned} X_{n+1} &= A_{n+1}e^{i\theta(n+1)} + B_{n+1}e^{-i\theta(n+1)} \\ &= A_ne^{i\theta(n+1)} + B_ne^{-i\theta(n+1)} + \Delta A_ne^{i\theta(n+1)} + \Delta B_ne^{-i\theta(n+1)}. \end{aligned} \tag{18}$$

In Eq. 18, the following condition can be imposed

$$\Delta A_ne^{i\theta(n+1)} + \Delta B_ne^{-i\theta(n+1)} = 0. \tag{19}$$

Then

$$X_{n+1} = A_n e^{i\theta(n+1)} + B_n e^{-i\theta(n+1)}. \tag{20}$$

From Eq. 20,

$$\begin{aligned} X_{n+2} &= (A_n + \Delta A_n)e^{i\theta(n+2)} + (B_n + \Delta B_n)e^{-i\theta(n+2)} \\ &= A_n e^{i\theta(n+2)} + B_n e^{-i\theta(n+2)} + \Delta A_n e^{i\theta(n+2)} + \Delta B_n e^{-i\theta(n+2)}. \end{aligned} \tag{21}$$

Introducing Eqs. 16, 20 and 21 into Eq. 13 and invoking the fact that Eq. 15 is a solution of Eq. 13*a*, the following relationship is obtained:

$$\Delta A_n e^{i\theta(n+2)} + \Delta B_n e^{-i\theta(n+2)} = \mu f(X_{n+1}, X_n). \tag{22}$$

ΔA_n and ΔB_n can be solved from Eqs. 19 and 22. The solutions are guaranteed due to the fact that $e^{i\theta n}$ and $e^{-i\theta n}$ are linearly independent of each other. Its Wronskian will not vanish.

$$\begin{aligned} \Delta A_n &= -\mu f(X_{n+1}, X_n)e^{-i\theta(n+1)}/e^{-i\theta} - e^{i\theta} \\ &= l\mu e^{-i\theta(n+1)} f(A_n e^{in\theta} + B_n e^{-in\theta}, A_{n+1}e^{i(n+1)\theta} + B_{n+1}e^{-i(n+1)\theta}) \end{aligned} \tag{23}$$

$$\begin{aligned} \Delta B_n &= -l\mu e^{i(n+1)\theta} f(X_{n+1}, X_n) \\ &= -l\mu e^{i(n+1)\theta} f(A_n e^{in\theta} + B_n e^{-in\theta}, A_{n+1}e^{i(n+1)\theta} + B_{n+1}e^{-i(n+1)\theta}) \end{aligned} \tag{24}$$

where

$$l = 1/e^{i\theta} - e^{-i\theta} \qquad \theta \neq 0. \tag{25}$$

Equations 23 and 24 are two first-order difference equations in terms of A_n, A_{n+1}, B_n, and B_{n+1}. It is seen that a second-order non-linear difference equation is reduced to two first-order ones. As the art of solving first-order difference equations is more complete (5), it is often easier to obtain an exact solution this way. More important than this is the fact that it opens a way to effect an approximation process, which will be discussed in the next section.

4. METHOD OF FIRST APPROXIMATION

In order to effect the approximation process, a way has to be found to expand uniquely an arbitrary function $\phi(n)$ of integral argument, n, defined when $n_1 \leqslant n \leqslant n_2$ as a linear function of $y_1, y_2, \cdots y_{n_2-n_1+1}$, where $y_1 \cdots y_{n_2-n_1+1}$ are orthogonal in the sense that

$$\sum_{n=n_1}^{n_1} g(n-1)y_j(n)y_k(n) = 0, \qquad \text{with} \qquad j \neq k.$$

One such development (6) is shown as follows

$$\phi(n) = \tfrac{1}{2}a_0 + \sum_{m=1}^{k} \left(a_m \cos n\frac{2\pi m}{w} + b_m \sin n\frac{2\pi m}{w}\right). \tag{26}$$

Where $w = 2k + 1$,

$$\phi(n) = \tfrac{1}{2}a_0 + \tfrac{1}{2}a_{k+1}\cos \pi n + \sum_{m=1}^{k}\left(a_m \cos n\frac{2\pi m}{w} + b_m \sin n\frac{2\pi m}{w}\right), \quad (27)$$

and where $w = 2k + 2$,

$$a_m = \frac{2}{w}\sum_{r=0}^{w-1}\phi(r)\cos m\frac{2\pi r}{w} \quad (28)$$

$$b_m = \frac{2}{w}\sum_{r=0}^{w-1}\phi(r)\sin m\frac{2\pi r}{w} \qquad (m = 0, 1, \cdots k + 1) \quad (29)$$

Equations 23 and 24 can be written as

$$\Delta A_n = \mu f_1(n, A, B, \theta) \quad (30)$$

$$\Delta B_n = \mu f_2(n, A, B, \theta). \quad (31)$$

Due to the presence of the small parameter, μ, on the right hand sides of Eqs. 30 and 31, A and B vary slowly. It can be approximated that they remain constants in these expressions.

If $2\pi/\theta = C$, where C is an integral constant, $f_1(n, A, B, \theta)$ and $f_2(n, A, B, \theta)$ will be periodic functions of n with period C. If C fails to be an integral constant, a close approximation can be achieved.

By the above argument, $f_1(n, A, B, \theta)$ and $f_2(n, A, B, \theta)$ of Eqs. 30 and 31 can be considered exactly or approximately as periodic functions of period w.

Due to this property, by using Eqs. 26 or 27, 28 and 29, Eqs. 30 and 31 can be written as

$$\begin{aligned}\Delta A_n &= \mu\left[\tfrac{1}{2}a_{01} + \sum_{m_1=1}^{k}\left(a_{m_1}\cos m_1\frac{2\pi n}{w} + b_{m_1}\sin m_1\frac{2\pi n}{w}\right)\right]\\ \Delta B_n &= \mu\left[\tfrac{1}{2}a_{02} + \sum_{m_2=1}^{k}\left(a_{m_2}\cos m_2\frac{2\pi n}{w} + b_{m_2}\sin m_2\frac{2\pi n}{w}\right)\right].\end{aligned} \quad (32)$$

Where $w = 2k + 1$,

$$\begin{aligned}\Delta A_n = \mu\Bigg[&\tfrac{1}{2}a_{01} + \tfrac{1}{2}a_{k+1,}\cos \pi n\\ &+ \sum_{m_1=1}^{k}\left(a_{m_1}\cos m_1\frac{2\pi n}{w} + b_{m_1}\sin m_1\frac{2\pi n}{w}\right)\Bigg]\\ \Delta B_n = \mu\Bigg[&\tfrac{1}{2}a_{02} + \tfrac{1}{2}a_{k+1,}\cos \pi n\\ &+ \sum_{m_2=1}^{k}\left(a_{m_2}\cos m_2\frac{2\pi n}{w} + b_{m_2}\sin m_2\frac{2\pi n}{w}\right)\Bigg],\end{aligned} \quad (33)$$

and where $w = 2k + 2$,

$$a_{m_{1,2}} = \frac{2}{w} \sum_{r=0}^{w-1} f_{1,2}(r) \cos m_{1,2} \frac{2\pi r}{w}$$

$$b_{m_{1,2}} = \frac{2}{w} \sum_{r=0}^{w-1} f_{1,2}(r) \sin m_{1,2} \frac{2\pi r}{w} \qquad m = 0, 1, \cdots k + 1.$$

At this stage, the first approximation process (1, 2), which has been widely used in solving non-linear differential equations, is introduced.

Sum both sides of Eq. 32 from $n = N$ to $n = N + w - 1$, where N is a positive integer and consider that $a_{m_{1,2}}$ and $b_{m_{1,2}}$ remain constant in this interval, we can obtain

$$\Delta A_n = \frac{\mu}{2} a_{01} \tag{34}$$

$$\Delta B_n = \frac{\mu}{2} a_{02} \tag{35}$$

where

$$\begin{aligned} a_{01} &= \frac{2}{w} \sum_{r=0}^{w-1} f_1(r, A, B) \\ a_{02} &= \frac{2}{w} \sum_{r=0}^{w-1} f_2(r, A, B). \end{aligned} \tag{36}$$

It can be seen that Eqs. 23 and 24 are first-order equations with variable coefficients, while Eqs. 34 and 35 clearly indicate that they are a set of difference equations of first order with constant coefficients, which are more susceptible to solutions.

5. DISCUSSION

In studying the performance of a physical system, it is often desirable to obtain the amplitude, K, and phase displacement, ψ, of possible sustained oscillations.

From Eq. 16,

$$\begin{aligned} X_n &= A(n)e^{in\theta} + B(n)e^{-in\theta} \\ &= a(n) \cos n\theta + b(n) \sin n\theta \\ &= K \sin (n\theta + \psi) \end{aligned} \tag{37}$$

where

$$a(n) = A(n) + B(n) = \text{a real function} \tag{38}$$

$$b(n) = i[A(n) - B(n)] = \text{a real function} \tag{39}$$

$$K = \sqrt{a^2 + b^2} = \sqrt{4A(n)B(n)} \tag{40}$$

$$\psi = \tan^{-1}\frac{a}{b} = \tan^{-1}\frac{A(n) + B(n)}{i[A(n) - B(n)]}. \tag{41}$$

Solving Eqs. 38 and 39 for $A(n)$ and $B(n)$,

$$B(n) = \tfrac{1}{2}[a(n) + ib(n)] \tag{42}$$

$$A(n) = \tfrac{1}{2}[a(n) - ib(n)]. \tag{43}$$

By the use of Eqs. 42 and 43, Eqs. 34 and 35 can be transferred to another set of difference equations of real variables, a and b, that is,

$$\Delta a_n = \mu a_{01}'(a, b) \tag{44}$$

$$\Delta b_n = \mu b_{02}'(a, b) \tag{45}$$

From Eq. 40, it is seen that

$$\begin{aligned} K^2(n) &= a^2(n) + b^2(n) \\ \Delta K^2(n) &= \Delta[a^2(n) + b^2(n)] \\ 2\Delta K(n)\cdot K(n) + (\Delta K)^2 &= 2a\Delta a + 2b\Delta b + (\Delta a)^2 + (\Delta b)^2. \end{aligned} \tag{46}$$

Due to the presence of μ in Eqs. 44 and 45, $(\Delta a)^2$, $(\Delta b)^2$, and subsequently $(\Delta K)^2$, in Eq. 46 can be neglected:

$$\begin{aligned} K\Delta K(n) &= a\Delta a + b\Delta b \\ &= \mu[aa_{01}'(a, b) + ba_{02}'(a, b)], \end{aligned} \tag{47}$$

since

$$\begin{aligned} a &= K\sin\psi \\ b &= K\cos\psi. \end{aligned} \tag{48}$$

Substituting Eq. 48 into Eq. 47,

$$\Delta K(n) = \frac{\mu}{K(n)} F(K, \psi). \tag{49}$$

Equation 49 will reveal the variation of the amplitude of a system response due to any given initial conditions. Besides, it will show the recurrent perturbation of the amplitude of sustained oscillation. If there is any approximately fixed oscillation amplitude, it can be found by solving $F(K, \psi) = 0$.

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A SIMPLE UNIFIED FIELD EQUATION AND HYPOTHESIS

BY

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ABSTRACT

A unified field equation of force, that is, $F_u = m_e^3c^3/h\alpha_e^2 = m_e^3c^4\epsilon_0/e^2\alpha_e = 0.5\ m_e^3c^5/E_R\hbar = \hbar cS_e^3/M_b^3m_e^2 = hc/L_m^3 = mr_e^3\omega_0^3\omega_e^3/r_e^3B_a^3 = mr_e\omega_e^3\omega_n^3/\omega_p^3B_a^3 = G/\mu_b^3 = 1.23163 \times 10^{23}$ g cm/sec^4 gauss^2 $= 3.981 \times 10^{8}$ dynes has been developed from well-known physical quantities such as electronic mass (m_e); the light velocity constant (c); the angular momentum constant ($\hbar = h/2\pi$); the electronic fine structure constant (α_e); the dielectric constant ($\epsilon_0 = 1.0$); the electronic charge (e); the Rydberg energy (E_R); electronic spin ($\hbar/2$); the Bohr magneton (M_b); the electron's "classical radius" (L_m); the Earth's mass (m); the moon's average orbital radius (r_e) and angular velocity (ω_e); the rotating Earth's angular velocity (ω_e), radius (r_e), polar magnetic intensity (B_a), angular velocity of nutation (ω_n), angular velocity of precession (ω_p); the gravitation constant (G); and a proposed proportionality constant of terrestrial and stellar magnetism ($\mu_b = r_eB_a/m\omega_e = 7.359 \times 10^{-18}$ cm sec gauss/g $= 1.2943 \times 10^{-8}$ cm/g, where one gauss unit is considered equal to 1.7589×10^{7}/sec). The fact that $\hbar c\mu_b^2/GL_m^2$ is equal to 1.000 suggests that μ_b and L_m are, respectively, universal constants of macroscopic and submicroscopic particle magnetism, and that the equation F_u prescribes a single universal physical law governing the action-reaction of all particles and fields, whether astronomic or submicroscopic in character.

SUMMARY

A unified field equation of force, $F_u = G/\mu_b^2 = \hbar c/L_m^2 = 1.23163 \times 10^{23}$ g cm/sec^4 gauss^2 $= 3.981 \times 10^{8}$ dynes, is proposed to correlate the gravitational (G) and magnetic (μ_b) fields of macroscopic particle physics with the precisely evaluated quantum ($\hbar$ and c) and magnetic (L_m) fields of submicroscopic particle physics. The fact that $\hbar c\mu_b^2/GL_m^2 = 1.000$ then suggests the unification and/or the existence of a single physical law which may govern the action-reaction of all particles and fields, that is, whether characterized as classical mechanical, quantum mechanical, electromagnetic, nuclear, mesonic, etc.

The constant $\mu_b = r_eB_a/m\omega_e \cong M/S = 7.359 \times 10^{-18}$ cm sec gauss/g $= 1.2943 \times 10^{-8}$ cm/g has been shown to prescribe a quite precise universal relationship between a macroscopic body's equatorial radius r_e, polar intensity of magnetization B_a, mass m, and equatorial angular velocity ω_e, and/or a proportionality between its magnetic moment M and "spin" S, for example, where $M \cong r_e^2r_eB_a$ and $S \cong mr_e^2\omega_e$, and where r_e is the axial radius. The constant $L_m = rB/\omega = mM/S = 1.60206 \times 10^{-20}$ cm sec gauss $= 2.81787 \times 10^{-13}$ cm is believed to prescribe similar relationships and proportionalities between properties of the submicroscopic particles of physics. (Note that the alternate dimensions of L_m, μ_b, and F_u reflect the proposed equivalence, one gauss $= 1.7589 \times 10^{7}$ sec^{-1}.)

It has also been shown that F_u is equal to $m_e^3c^3/h\alpha_e^2$ or $m_e^3c^4\epsilon_0/e^2\alpha_e$ or $\hbar cS_e^3/M_b^3m_e^2$ or $0.5\ m_e^3c^5/E_R\hbar$, which affords correlations of G, μ_b, L_m, $\hbar$, and c with the specific electronic qualities of mass m_e, charge e, fine structure constant α_e, spin S_e (equal to $\hbar/2$), magnetic moment M_b (Bohr magneton), Rydberg energy E_R, and the dielectric constant $\epsilon_0 = 1.0$.

[1] Oak Ridge, Tenn.

The simplicity and "reality" of the cgs force equation may be illustrated in the well-known case of the Earth, for example, where $F_u = G/\mu_b^2 = mr_e^3\omega_e^2\omega_n^2/r_e^3B_a^2 = mr_e\omega_e^3\omega_n^2/\omega_p^2B_a^2$ is found to correlate the moon's orbital radius r_e and angular velocity ω_e with the rotating Earth's mass m, radius r_e, equatorial angular velocity ω_e, angular velocity of precession ω_p, angular velocity of nutation ω_n, and polar intensity of magnetization B_a. Since most of the physical phenomena and motions of rotating, revolving, and interacting bodies are determinable from such basic properties of mass, space, time^{-1}, and/or magnetization, it is hypothesized that essentially all actions and reactions of particle systems and fields may conform to the universal physical law (or laws) inherent in $F_u = G/\mu_b^2 = \hbar c/L_m^2$.

For convenience in checking the mathematics and dimensions of the above F_u equation, the following values are supplied:

$$G = 6.67 \times 10^{-8}\,\frac{\text{dyne cm}^2}{\text{g}^2} = 6.67 \times 10^{-8}\,\frac{\text{cm}^3}{\text{g sec}^2} = \frac{r_e^3\omega_e^2}{m}$$

$$\mu_b = 7.359 \times 10^{-16}\,\frac{\text{cm sec gauss}}{\text{g}} = 1.2943 \times 10^{-8}\,\frac{\text{cm}}{\text{g}} = \frac{r_eB_a}{m\omega_e}$$

$$\hbar = 1.0544 \times 10^{-27}\,\text{erg}\cdot\text{sec} = 1.0544 \times 10^{-27}\,\frac{\text{g cm}^2}{\text{sec}}$$

$$c = 2.9977 \times 10^{10}\,\frac{\text{dyne}\cdot\text{sec}}{\text{g}} = 2.9977 \times 10^{10}\,\frac{\text{cm}}{\text{sec}}$$

$$L_m = 2.81787 \times 10^{-13}\,\text{cm} = 1.60206 \times 10^{-20}\,\text{cm sec gauss}$$

$$m_e = 9.108 \times 10^{-28}\,\text{g}$$

$$\alpha_e = 7.29729 \times 10^{-3}\,\frac{\text{sec}}{\text{sec}} = 4.1488 \times 10^{-10}\,\text{gauss}\cdot\text{sec}$$

$$\frac{e^2}{\epsilon_0} = 2.30675 \times 10^{-19}\,\frac{\text{g cm}^3}{\text{sec}^2} = 1.3115 \times 10^{-26}\,\frac{\text{g cm}^3\,\text{gauss}}{\text{sec}}$$

$$S_e = 5.272 \times 10^{-28}\,\text{dyne cm sec} = 5.272 \times 10^{-28}\,\frac{\text{g cm}^2}{\text{sec}}$$

$$M_b = 9.2837 \times 10^{-21}\,\text{cm}^3\,\text{gauss} = 1.63291 \times 10^{-12}\,\frac{\text{cm}^3}{\text{sec}}$$

$$E_R = 13.597\,\text{ev} = 2.1796 \times 10^{-11}\,\frac{\text{g cm}^2}{\text{sec}^2}$$

$$m = 5.98 \times 10^{27}\,\text{g} = 6.6 \times 10^{22}\,\text{tons}$$

$$r_e = 3.84 \times 10^{10}\,\text{cm} = 2.38 \times 10^5\,\text{miles}$$

$$\omega_o = 2.66 \times 10^{-6}\,\frac{\text{rad}}{\text{sec}} = \frac{1.0\,\text{rev}}{27.33\,\text{days}}$$

$$\omega_e = 7.29 \times 10^{-5}\,\frac{\text{rad}}{\text{sec}} = \frac{1.0\,\text{rev}}{\text{day}}$$

$$r_e = 6.36 \times 10^8\,\text{cm} = 3963\,\text{miles}$$

$$B_a = 0.5\,\text{gauss} = \frac{8.86 \times 10^8}{\text{sec}}$$

$$\omega_n = 1.07 \times 10^{-9}\,\frac{\text{rad}}{\text{sec}} = \frac{1.0\,\text{rev}}{18.6\,\text{years}}$$

$$\omega_p = 7.65 \times 10^{-12}\,\frac{\text{rad}}{\text{sec}} = \frac{1.0\,\text{rev}}{26.000\,\text{years}}$$

INTRODUCTION

The search for a unified field theory of physics has occupied the attention of theoreticians throughout the history of science. In a way, the revelation of such a quantitative theory or law has been considered as the ultimate aim of all investigations into the understanding of nature, matter, and the universe itself.

It has been said that science assumes nature is unified; and that without a simple faith in the existence of only one set of natural laws, a single plan for the universe, science as we know it could not exist. However, modern knowledge of the myriad of complexities and uncertainties in the physics of the submicroscopic world of particles and waves has led many scientists to believe that a valid unified law is probably an unattainable goal, or perhaps an idle dream of the philosopher.

In any event, the complicated problem of discovering some single physical law or correlation, which is equally applicable to both the classical mechanics of the macroscopic world and the quantum- or electro-mechanics of the submicroscopic world, is sufficiently abstruse to discourage most from devoting serious effort to its resolution. Perhaps the best known attempt to discover such a law was by Dr. Albert Einstein in his search for a general theory of relativity. He maintained that some single correlation, probably based on simple realities, might someday be established to unify the discordances of physical principles now believed to govern separately behavior of the fields (and particles) of gravitation, electromagnetism, and quantum mechanics. The success of his special theory of relativity in refining subtle discrepancies in the laws of gravitation led the scientific world to hope that his ambitious goal might be attained. There were few modern scientists, however, who believed that such a law could ever be characterized by simplicity or reality.

It was believed that any theory or law of such magnificent generalization (that is, to embrace simultaneously the cosmic vastness of mass, space, and time as well as the microscopic minuteness of mass, space, and time) might afford precise refinements of many physical laws which just miss exactness, as well as possible explanations of mysterious unseeable phenomena evidenced experimentally in atomic and nuclear science. For examples, a truly valid unified law might permit resolution of imperfections in the gas laws; correlations of the electromagnetic and other properties of solids, liquids, and gases; revelation of the precise nature of atomic, nuclear, and mesonic binding forces; explanation of the observed great differences (ca. 10^{40}) of interaction forces between particles in macroscopic and microscopic physics; and, in general, more quantitative insight into every known phase of scientific and technologic endeavor.

With such a formidable description of a unified law of nature, it is understandable that the presently proposed candidate must be con-

sidered only as a modest hypothesis, a possible step toward the ultimate attainment of complete generalization. Some background for the presentation has been published previously (1, 2, 3).[2]

One specific novel aspect of the study has been the intriguing revelation that the well-known properties of the Earth's solid rotating body (for example, its 6.6×10^{21} tons of mass, its 3963-mile radius, its one revolution per day on its axis, its 0.5-gauss polar magnetic field, its 26,000-year period of precession, and its 18.6-year period of nutation) may be rather precisely related to the most fundamental constants (and/or laws) of physics, such as G, $\hbar$, and c, as well as electromagnetic quantities such as the electron's mass, charge, fine structure constant, magnetic moment, etc. Such a revelation, perhaps reminiscent of Kepler's discovery of the harmonic motions of planetary systems and Newton's eventual disclosure of the gravitational laws, is considered to hold the key to an "explanation" of the Earth's magnetic field, and of the magnetic fields of other massive bodies. In a way, the knowledge of such quantitative correlations is employed in conjunction with precise knowledge of properties of submicroscopic particles (such as the electron) to suggest that considerable "proofs" of the proposed unified field equation already exist in well-established scientific data.

DISCUSSION

It may be believed that the reason for previous failures to discover a quantitative unified field theory or law is connected with the fact that science hasn't yet resolved the mysteries of "field"—or "particle"—magnetism, in either the (1) classical mechanical (macroscopic) or the (2) quantum mechanical (microscopic) systems of physics. In the present case, it is assumed that these two major mysteries have now been resolved by precise universal laws inherent in two "new" fundamental physical constants:

$$\mu_b = \left(\frac{G\hbar\alpha_e^2}{m_e^2c^2}\right)^{1/2} = \left(\frac{G\alpha_e e^2}{m_e^2c^4\epsilon_0}\right)^{1/2} = 1.2943 \times 10^{-8}\ \frac{\text{cm}}{\text{g}}$$

$$= 7.3586 \times 10^{-16}\ \frac{\text{cm sec gauss}}{\text{g}}$$

$$L_m = \frac{\hbar\alpha_e}{m_e c} = \left(\frac{\hbar c\alpha_e e^2}{m_e^2c^4\epsilon_0}\right)^{1/2} = 2.81787 \times 10^{-13}\ \text{cm}$$

$$= 1.60206 \times 10^{-20}\ \text{cm sec gauss.}$$

(The alternate dimensional expression reflects the use of a vital assumption or revelation, discussed previously (1, 2, 3), concerning the units of magnetic intensity, that is, that 1.0 gauss = 1.7589×10^7

[2] The boldface numbers in parentheses refer to the references appended to this paper.

$\sec^{-1}$.) The classical mechanical constant μ_b of magnetic permeability, despite its complex appearance, is believed to define a simple proportionality of properties and actions of macroscopic (or astronomic) solid bodies, that is,

$$\mu_b \cong \frac{\text{radius} \times \text{polar magnetic intensity}}{\text{mass} \times \text{angular velocity of axial rotation}}$$

$$\cong \frac{\text{magnetic moment}}{\text{spin}}$$

whereas the quantum mechanical constant L_m of specific magnetic permeability is believed to define a somewhat similar proportionality of properties and actions of microscopic particles (or particle-waves), that is,

$$L_m \cong \frac{\text{radius} \times \text{polar magnetic intensity}}{\text{angular velocity of axial rotation}}$$

$$\cong \frac{\text{mass} \times \text{magnetic moment}}{\text{spin}}.$$

In order to construct a simple unified field theory and equation it is proposed that knowledge afforded by these two universal constants or laws be added to the already existing knowledge afforded by the three famous universal laws or constants:

$$G = 6.67 \times 10^{-8} \frac{\text{dyne cm}^2}{\text{g}^2} = 6.67 \times 10^{-8} \frac{\text{cm}^3}{\text{g sec}^2}$$

$$\hbar = 1.0544 \times 10^{-27} \text{ erg} \cdot \text{sec} = 1.0544 \times 10^{-27} \frac{\text{g cm}^2}{\text{sec}}$$

$$c = 2.9977 \times 10^{10} \frac{\text{dyne} \cdot \text{sec}}{\text{g}} = 2.9977 \times 10^{10} \frac{\text{cm}}{\text{sec}}.$$

The first of these, the gravitational constant G, was revealed by Newton after Kepler had observed the harmonic qualities of the planetary orbits of astronomy. As with μ_b and L_m above, it was noted to represent a constant of proportionality existing between properties and actions of the macroscopic bodies of mass, for example,

$$G \cong \frac{(\text{orbital radius})^3 \times (\text{angular velocity})^2}{\text{mass of central body}}$$

$$\cong \frac{\text{satellite's force} \times (\text{orbital radius})^2}{\text{mass of satellite} \times \text{mass of central body}}.$$

It thus became a basis for Newton's several universal laws of gravitation and gravity which set the stage for the rapid development of classical physics and the mechanical age of science and technology.

The second quantity, the constant of "angular momentum" $\hbar$ (an outgrowth of the "constant h of action"), was revealed by modern theoreticians after Planck had observed the harmonic mathematical and dimensional relations between the energy and frequency of electromagnetic waves ($E = h\nu$), and after deBroglie had observed similar harmonic relations of waves created by microscopic particles of matter ($\lambda = h/mv$). As with μ_b, L_m, and G above, it was implied that it represented a constant of proportionality existing between properties and actions of microscopic particles of mass, for example,

$$\hbar \cong \text{mass} \times (\text{effective radius})^2 \times \text{angular velocity}$$
$$\cong (\text{mass} \times \text{energy} \times \text{effective radius}^2)^{1/2}.$$

It, along with the companion constant h or $2\pi\hbar$, became the basis for development of the several universal laws of radiation, atomic physics, and quantum mechanics, and thus the modern age of science and technology.

The last quantity, the constant of light velocity c, was revealed by Einstein to have major theoretical implications as a limiting measure of motion, after predecessors had observed that photons or light waves of different energy-content moved with the identical velocity c. As with μ_b, L_m, G, and $\hbar$ above, it was implied that it represented a constant of proportionality between properties and actions of either microscopic or macroscopic particles of mass, for example,

$$c \cong \text{minimum radius} \times \text{maximum angular velocity}$$
$$\cong \left(\frac{\text{intrinsic energy}}{\text{particle mass}}\right)^{1/2}.$$

In a way, it was presumed to represent inherently a unified field equation, since it was the first law believed applicable to either the classical mechanical or quantum mechanical fields and particles. Indeed, it formed the basis for the development of the refined laws or principles of Einstein's special relativity of classical mechanical systems, as well as his attempt at complete generalization (to include quantum mechanical systems), that is, general relativity. Its knowledge, as $c = (\text{energy/mass})^{1/2}$, thus set the stage for development of nuclear science and the modern searches into meson, cosmic ray, and high-energy physics.

Before proceeding to discussions of the simple unified field equation, there are three additional points which should be treated briefly. First,

the existence of more than one universal constant or law implies that various combinations and permutations of the basic constants represent additional constants of specific value to theoretical interpretations and correlations of the properties and actions of particles of fields. One such combination is

$$\hbar c = 3.1611 \times 10^{-17}\ \text{erg}\cdot\text{cm} = 3.1611 \times 10^{-17}\ \frac{\text{g cm}^3}{\text{sec}^2}$$

which is considered in this study to be a significant constant of proportionality existing between properties and actions of microscopic particles or particle-waves, that is,

$$\hbar c \cong \frac{\text{particle energy} \times \text{associated photon wave radius}}{2}$$

$$\cong \frac{\text{particle energy} \times (\text{effective radius})^2}{\text{axial radius}}.$$

(The "axial radius" in the last expression may be considered as a known quantity if the particle's "spin" is known, that is, it is equal to the product of "spin" and c, divided by the intrinsic or mc^2 energy of the particle.)

Secondly, the mysterious and major differences in the laws of classical mechanics (such as gravitational binding energy $= Gm_1m_2/r$) and the laws of quantum or nuclear mechanics (such as particle binding energy $= \hbar^2/mr^3$) have suggested the somewhat strange possibility that the mass-particles of nature fall into two quite distinct classes, which obey similarly distinct and separate mechanical laws of behavior. The distinction seems to be further accentuated by certain hypotheses that astronomic or classical mechanical particles may, at high spin velocities, approach a limiting ratio of radius-to-mass (and/or maximum density) equal to Gc^{-2}, whereas other hypotheses suggest that microscopic or quantum mechanical particles may similarly approach a limiting radius $\times$ mass product (and/or maximum density) equal to $\hbar c^{-1}$. On the one hand, the rather weakly-bound masses of macroscopic bodies may thus appear to be disruptible by centrifugal forces at high peripheral spin velocities to cause fission or degeneration into smaller and denser particles; whereas, on the other hand, it would appear that the very strongly-bound masses of microscopic bodies (and associated matter-waves) may actually fuse (or increase in mass) on approach to the light velocity limit. The simultaneous existence of two such antithetical particle systems of physics has effectively confounded science philosophers and threatened the peaceful co-existence of the classical and quantum mechanical branches of physics. Most often, there has existed the belief that one or the other specific systems of physics must be in

error, or perhaps misinterpreted. Such beliefs led to spirited debates between the older adherents of classical (or "causalistic") and the modern adherents of the quantum (or "indeterministic") branches of theory. In a way, such differences constitute the reasons behind (*a*) the need for a unified field law and (*b*) the usually uncomprising belief that classical and quantum laws can never be reconciled into one rational, all-embracing, law.

Thirdly, it will have been observed that above discussions of the universal constant G, $\hbar$, and c did not include reference to the highly important electronic or electromagnetic theories and constants. In actuality, as evidenced in previous (1, 2, 3) studies and in the above expressions for the constants μ_b and L_m, attention to such theories and constants has formed the major basis for development of the present so-called "simple unified field theory." Derivations of the magnetic constants μ_b and L_m are noted to relate these universal quantities to specific properties of the electron, such as its rest mass m_e, its fine structure or "coupling" constant α_e, its charge e or $(\hbar c\alpha_e)^{1/2}$ and the dimensionless dielectric constant $\epsilon_0 = 1.0$ of the vacuum. As a basic thesis of the present search for a unified field theory, it was assumed that the mathematical expedience of the latter quantity (and of the magnetic permeability constant $\mu_0 = 1.0$) had obscured, for over a century, the mechanical "realities" and interrelationships of electronic and magnetic phenomena. However, such an assumption was not rashly considered as a challenge to the magnificence of either Maxwell's equations of electricity and magnetism or the philosophical-mathematical reasons for choosing $\epsilon_0 = 1.0$ and $\mu_0 = 1.0$. Rather, it has been hypothesized or merely suggested (2) that modern knowledge permits the use of alternates:

$$\epsilon_g = \frac{m_e^2}{\hbar\alpha_e c} \cong \frac{\text{spin}}{\text{magnetic moment} \times c^2} = 3.595 \times 10^{-36} \frac{\text{g sec}^2}{\text{cm}^3}$$

$$= 6.313 \times 10^{-29} \frac{\text{g sec}}{\text{cm}^3 \text{ gauss}}$$

$$\mu_g = \frac{\hbar\alpha_e}{m_e^2 c} \cong \frac{\text{magnetic moment}}{\text{spin}} = 3.094 \times 10^{14} \frac{\text{cm}}{\text{g}}$$

$$= 1.7589 \times 10^{7} \frac{\text{cm sec gauss}}{\text{g}}$$

which have figured strongly in the revelations of the more general implications believed inherent in the "new" universal constants μ_b and L_m.

THE UNIFIED FIELD EQUATION

Considering the above universal laws or constants G, μ_b, and c of classical mechanics and the universal laws or constants $\hbar$, L_m, and c of

quantum mechanics, and the relative simplicity of the particle- and field-proportionalities which they individually define, it is proposed that certain of their specific combinations and permutations give rise to a simple unified field equation:

$$F_u = \frac{G}{\mu_b^2} = \frac{\hbar c}{L_m^2} = 3.981 \times 10^8 \frac{\text{g cm}}{\text{sec}^2} = 1.23163 \times 10^{23} \frac{\text{g cm}}{\text{sec}^4 \text{ gauss}^2}.$$

This constant or law of universal "force" is proposed to correlate precisely and perspicuously an essentially infinite variety of particle- or field-properties and actions of either classical mechanical physics (as $G\mu_b^{-2}$) or quantum-mechanical and electromagnetic physics (as $\hbar c L_m^{-2}$). In essence, it is considered to resolve the mystery of relationships which connect the magnetic phenomena (μ_b and L_m) of particles and fields to the mechanical and motion phenomena (G and $\hbar c$) of those particles and fields. Its nature suggests that, although the specific mechanical laws governing macroscopic bodies (G) and microscopic bodies ($\hbar c$) are distinctly different in quantity and quality, and although the specific magnetic laws governing macroscopic bodies (μ_b) and microscopic bodies (L_m) are distinctly different in quantity and quality, the "interaction" or resolution of such laws in the form of F_u is one and the same for either class of bodies, or for the "fields" of their action and reaction. This all-embracing unity can best be expressed by the obvious and simple relationship,

$$\frac{GL_m^2}{\hbar c \mu_b^2} = 1.0000.$$

Thus, F_u's dimensions would suggest that it may define a precise pattern or matter-structure of mass, space, and time^{-1} (and/or magnetic intensity) in universal order.

From what has been previously discussed, concerning the limits which the universal constant c impresses on either macroscopic or microscopic particles, it may naturally be presumed that the following constant, as well as F_u and c, somehow governs the limits of properties and actions of all particles and fields, that is,

$$P_u = \frac{Gc}{\mu_b^2} = \frac{\hbar c^2}{L_m^2} = 1.194 \times 10^{19} \frac{\text{g cm}^2}{\text{sec}^3} = 3.694 \times 10^{33} \frac{\text{g cm}^2}{\text{sec}^5 \text{ gauss}^2}.$$

Thus, a proportionality constant of universal "power" may also be considered to control the mass, space, and time^{-1} (and/or magnetic intensity) of particles and of fields. This illustrative point is made to suggest how a variety of combinations and permutations of the universal constants G, μ_b, $\hbar$, L_m, and c may become useful for precise interpreta-

tions of a large number of possibly unsuspected physical phenomena in universe systems, whether macroscopic (or cosmologic) or microscopic (or submicroscopic) in nature. Indeed, as previously (3) discussed, there may possibly be some tens of thousands of useful combinations and permutations of the specific "universal units"

$$m = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.17699 \times 10^{-5} \text{ g (mass)}$$

$$r = \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.61562 \times 10^{-33} \text{ cm (radius, or length)}$$

$$\omega = \left(\frac{c^5}{G\hbar}\right)^{1/2} = 1.85559 \times 10^{43} \frac{\text{radians}}{\text{sec}} \text{ (angular velocity or time}^{-1}\text{)}$$

$$B = \left(\frac{L_m c^4}{G\hbar}\right)^{1/2} = \frac{3.2364 \times 10^{61}}{\text{sec}} = 1.84253 \times 10^{56} \text{ gauss}$$

(magnetic intensity).

(In the interest of geometrical, wave-mechanical, and thermodynamic considerations, the two additional quantities π and "unit temperature" $T = (\hbar c^5/4Gk^2)^{1/2} = 7.08681 \times 10^{31}$ °A are also recognized as valuable constants.)

However, for the present brief discussion, some specific amplification concerning the nature and inherent qualities of the "force" constant F_u will be presented.

SOME CHARACTERISTICS OF THE UNIVERSAL FORCE CONSTANT AND EQUATION

Previous "derivations" of the constants or laws of μ_b and L_m from well-known constants of electricity and magnetism make it obvious that the general law of F_u may be similarly derived to show its precise connection to electromagnetic phenomena and fields, for example,

$$F_u = \frac{m_e^2 c^3}{\hbar \alpha_e^2} = \frac{m_e^3 c^4 \epsilon_0}{\alpha_e e^2} = \frac{\hbar c S_e^2}{m_e^3 M_b^2} = \frac{2E_R m_e c}{\hbar \alpha_e^4} = \frac{\epsilon_0 c^4}{\alpha_e} = \frac{c^2}{\mu_0 \alpha_e^4} = \frac{G}{\mu_b^2} = \frac{\hbar c}{L_m^2}$$

which illustrates its relationship to the electron spin S_e (equal to $\hbar/2$), the Bohr magneton M_b of electron magnetic moment, and the "Rydberg energy" E_R, as well as the previously discussed electronic quantities m_e, α_e, e, ϵ_0, ϵ_0 and μ_0. The last two expressions in the equation were included to illustrate how each of the several fundamental electromagnetic constants or laws can be readily expressed as functions of the gravitational constant G, the classical "magnetic permeability constant" μ_b, the quantum "specific magnetic permeability constant" L_m, etc. Such equalities afford a direct basis for perspicuous speculations concerning

the subtle interrelationships and philosophic connections between electromagnetic, electrostatic, quantum, nuclear, gravitational, and terrestrial (or stellar) magnetic fields. Knowledge of such mysterious connections has historically been considered as vital or mandatory in the search for the exotic secrets of levitation or anti-gravity forces, and of control over nuclear forces, mesonic forces, and of the forces which account for high-energy cosmic rays, strange cosmical phenomena, etc. Though certainly the simply stated equalities do not directly define answers to such complicated and subtle questions, it can be believed they may help define the possibility or probability of many heretofore unsuspected qualities of matter and of the complex interactions of fields.

In conjunction with a previous (2) study of the magnetic permeability constant $\mu_0 = \hbar\alpha_e(m_e^2c)^{-1}$ and the dielectric constant $\epsilon_0 = m_e^2(\hbar c\alpha_e)^{-1} = m_e^2\epsilon_0 e^{-2}$, it has been possible to dimensionalize the constant F_u in terms of a variety of units, for example, to include the four-unit systems of practical electricity, electromagnetism, magnetostatics, and electrostatics. The conversion to such "word" units, although not believed to be fundamentally important, may be somewhat helpful in providing a mechanism for the transition of thought from the established practical terminology to that which seemed to be required in the present search for unification of quantum, electromagnetic, and gravitational laws. Examples of such conversions are given below:

Practical Electrical Units

$$F_u = 7.002 \times 10^{16}\,\frac{\text{amp}\cdot\text{cm}}{\text{sec}} = 7.002 \times 10^{23}\,\frac{\text{g cm}^3}{\text{sec}^4\,\text{volt}}$$

$$= 1.23163 \times 10^{32}\,\frac{\text{cm}^3}{\text{sec}^3\,\text{ohm}} = 3.981 \times 10^{8}\,\frac{\text{g cm}}{\text{sec}^2}$$

Electromagnetic Units

$$F_u = 7.002 \times 10^{16}\,\frac{\text{abamp}\cdot\text{cm}}{\text{sec}} = 1.23163 \times 10^{23}\,\frac{\text{cm}^3\,\text{oersted}^2}{\text{g}}$$

$$= 1.23163 \times 10^{32}\,\frac{\text{cm}^3}{\text{sec}^3\,\text{abohm}} = 3.981 \times 10^{8}\,\frac{\text{g cm}}{\text{sec}^2}$$

Magnetostatic Units

$$F_u = 1.23163 \times 10^{23}\,\frac{\text{cm}^2}{\text{sec}^2\,\mu_0} = 7.002 \times 10^{16}\,\frac{\text{g cm}^3}{\text{sec}^3\,\text{maxwell}}$$

$$= 1.23163 \times 10^{32}\,\frac{\text{g cm}^5}{\text{sec}^4\,\text{maxwell}^2} = 3.981 \times 10^{8}\,\frac{\text{g cm}}{\text{sec}^2}$$

Electrostatic Units

$$F_u = 1.107 \times 10^{44} \frac{\text{cm}^4\ \epsilon_g}{\text{sec}^4} = 1.107 \times 10^{44} \frac{\text{statcoulomb}^2\ \text{cm}}{\text{g sec}^2}$$

$$= 2.099 \times 10^{26} \frac{\text{g cm}^3}{\text{sec}^4\ \text{statvolt}} = 3.981 \times 10^{8} \frac{\text{g cm}}{\text{sec}^2}$$

Though not specifically listed under either the electromagnetic or magnetostatic cases above, it will be remembered that the original derivation of the universal "force" constant resulted in "gauss" dimensions, that is

$$F_u = \frac{\hbar c S_e^2}{m_e^2 M_b^2} = 1.23163 \times 10^{23} \frac{\text{g cm}}{\text{sec}^4\ \text{gauss}^2}$$

where "spin" S_e is equal to $\hbar/2$, m_e is equal to 9.108×10^{-28} g, and M_b (the Bohr magneton) is equal to 9.2736×10^{-21} cm^3 gauss. Note that where the latter quantity is dimensionalized as 9.2736×10^{-21} erg·oersted^{-1}, F_u is equal to 1.23163×10^{23} cm^3 oersted^2 g^{-1}, as shown above under "electromagnetic units."

POSSIBLE USE OF THE FORCE EQUATION FOR INTERPRETATIONS OF BINDING FORCES

In classical mechanical or astronomic systems of particle interaction it is known that binding forces between mass centers and satellites may be calculated or closely approximated by use of the equation $F_s = Gm_cm_s/r_{cs}^2$. Similarly, in quantum mechanical or microscopic systems it has been suggested that binding forces between mass centers and satellites may be calculated or closely approximated by use of a different equation $F_s = \hbar^2/m_s r_{cs}^3$. In both systems, cases have been observed where small deviations from exactness have hinted that all factors affecting such binding forces are not accounted for in simple terms of G or $\hbar^2$. In the classical system, "special relativity" represented an attempt to correct such discrepancies, and in the quantum system a concept of "parity" and the assignment of "spin" (or half-integral adjustments in $\hbar$) seemed to be required. In such refinements of theory, it was in effect being realized that a unified field theory was needed, or that yet unrevealed principles were hampering the desired exactitude of knowledge concerning natural forces of interaction.

In a previous (1) study it was suggested that in classical systems, the interaction force might be alternately calculated or approximated by use of an equation $F_s = F_u L_s L_c/r_{cs}^2$, where F_u was the universal force constant and L was a quantity approximately equal to $\mu_s m_s$ or $\mu_b m_c$ (or definable as a quantity equal to the axial radius × the magnetic intensity/angular velocity, or proportional to the ratio of mass × magnetic moment/angular momentum). Similarly, in microscopic systems,

it appeared that the interaction force could be alternately calculated by $F_s = L_{mc}L_{ms}F_u/r_l r_{ec}$, or by $F_s = L_{mc}L_{ms}F_u r_{as}/r_{ec}^3$, where F_u and L_m were the universal constants, and where r_l was a photon wave radius or amplitude, r_{ec} was the effective radius or separation of particles and r_{as} was the axial or minimum radius of the satellite particle (equal to S_e/mc). Thus, the points were made that the constant or law F_u was employed in *both* the classical (macroscopic particles) and quantum (microscopic particles) systems of interaction, and that the magnetic properties (for example, magnetic moments or magnetic intensities) of the interacting particles, as well as their masses, were considered to be involved in the binding forces between the particles.

The suggested fundamental importance of the magnetic properties of particles on interaction forces and binding energies raises the mysterious question concerning the nature of magnetism and the processes which produce magnetization of mass-bodies. The following discussion, though not attempting actually to "explain" them, pursues these philosophic questions indirectly by illustrating some unusual correlations of the mechanical and magnetic properties of macroscopic bodies such as the Earth.

SIMPLE FORMULATIONS OF F_u AS A UNIFIED EQUATION OF PARTICLE PHYSICS

Previous studies (1, 2, 3) have presented quantitative examples showing how the constants such as $\hbar$, c, L_m, G, μ_b, and F_u may be precisely derived from observed data concerning the properties and motions of a body, such as the electron or the Earth. A review of the formulations may illustrate the simple "reality" of F_u as a particle-constant, and may suggest indirectly an imaginative mechanism for understanding its qualities as a field-constant or law.

In the specific case of the Earth, where rather accurate physical measurements have been possible, certain quantitative properties and motions may be well-defined in simple cgs dimensions. For example, the spherical solid body may be accurately described by a mass m (in g), an equatorial radius r_r (in cm), and an axial radius r_a (in cm). Owing to its gyro-like rotation in an east-west direction, it may be further described as having a rotational angular velocity ω_r (in radians·sec^{-1}), a nearly north-south or axially directed magnetic field of polar intensity B (in either gauss, maxwell cm^{-2}, or sec^{-1} units), a precessional angular velocity ω_p (in radians·sec^{-1}), and a nutational angular velocity ω_n (in radians·sec^{-1}). From the velocity of escape, it may also be described by a hypothetical "angular velocity of escape" ω_{es} (in radians·sec^{-1}), equal to $(Gm/r_a^3)^{1/2}$, perhaps related to its rotary motion at some time during the creation of its present geometry. Finally, in relation to the orbit of its satellite, the moon, it may be described by an associated orbital radius r_o (in cm) and an orbital angular velocity ω_o (in radians·

sec^{-1}). Thus one may perceive the simple "reality" of one property of mass, about three radial properties of space, and about six radial properties of time^{-1}, and/or one property of magnetization (for example, if in gauss units), all of which reflect forces which are acting (or which have acted) on the body. Many peculiar coincidences of relationship to the constants G, μ_b, c, and F_u have been noted, for example,

$$\frac{r_o^3\omega_o^2}{m} = \frac{r_r^3\omega_{ss}^2}{m} = 6.67 \times 10^{-8}\,\frac{\text{cm}^3}{\text{g sec}^2} = G$$

$$\frac{r_o B}{m\omega_r} = 1.294 \times 10^{-8}\,\frac{\text{cm}}{\text{g}} = 7.359 \times 10^{-18}\,\frac{\text{cm sec gauss}}{\text{g}} = \mu_b$$

$$\frac{r_r^2\omega_r^4\omega_n^5}{B\omega_p^6} = 9 \times 10^{20}\,\frac{\text{cm}^2}{\text{sec}^2} = c^2$$

$$\frac{r_r^2\omega_r^2\omega_n^3}{B\omega_p^3} = 5.1 \times 10^{0}\,\frac{\text{cm}^2}{\text{sec}^2} = 9.1 \times 10^{7}\,\frac{\text{cm}^2}{\text{sec}\cdot\text{gauss}} = \frac{G}{\mu_b}$$

$$\left(\frac{\omega_n\omega_r}{\omega_p^2}\right)^2 = 1.744 \times 10^{20} = \frac{\mu_b c^2}{G}$$

$$\frac{mr_o^3\omega_o^2\omega_r^2}{r_o^2B^2} = \frac{mr_r^2\omega_{ss}^2\omega_r^2}{r_o^2B^2} = \frac{mr_r^2\omega_r^3\omega_n^3}{r_oB^2\omega_p^2} = \frac{G}{\mu_b^2} = \frac{\hbar c}{L_m^2}$$

$$= \frac{m_e^2c^3}{\hbar a_e^2} = \frac{(m_ec^2)^2\epsilon_0}{\alpha_e e^2} = F_u.$$

It will be agreed that the occurrence of such a series of simple coincidences in nature would be an unlikely event. The fact that the properties of the best-known astronomic body can be so related to highly accurate electronic, radiation, or quantum mechanical quantities (as shown in the last equation above) would indeed involve a magnificent coincidence. The fact that the magnetic intensity B appears to be mysteriously related to specific properties of motion such as ω_r, ω_p, and ω_n poses an interesting philosophic question concerning the proposed inverse time (sec^{-1}) dimensions for that property, as previously discussed. As such, the measure of magnetization is suggested to represent a mysterious but finite resolution of gyroscopic motions which characterize a spinning body of mass.

In previous studies (1, 2) it was proposed that electromagnetic "word" units might be alternately expressed in terms of "mechanical equivalents" and dimensions, for example, current (amps) in g sec^{-1}, magnetizing force (oersteds) in g (cm sec)$^{-1}$, magnetic permeability (μ_0) in cm g^{-1}, magnetic intensity (gauss) in sec^{-1}, magnetic pole (maxwell) in cm^3 sec^{-1}, magnetic moment (maxwell·cm) in cm^3 sec^{-1},

etc. With this in mind, it is interesting to consider that the rotary motion of the Earth's mass generates a magnetizing force H equal to $m\omega_r r_r^{-1}$, with g (cm sec)$^{-1}$ dimensions and that it generates an axially directed magnetic field of intensity $B = \mu_b H$. Further it is interesting to consider that the Earth exhibits an angular momentum U proportional to $mr_r^2\omega_r$, a "spin" S proportional to $mr_e r_r \omega_r$, and a magnetic moment M proportional to $r_r^3 r_e B$, so that $M \cong \mu_b S$ or possibly $M \cong \mu_b U$. The Earth may also be characterized by a magnetic pole p which is proportional to $r_r^3 B$, a "dimensionless" fine structure or coupling constant α proportional (or equal) to $B\omega_r^{-1}$, a "charge" proportional to $(mr^3\omega_r B)^{1/2}$, etc. Though models of "reality" such as these have an unpopular or decadent flavor, all of the various coincidences would seem to strengthen the old hypothesis which asserted that connections exist between magnetization and angular momentum. Hopefully, the variety of revelations concerning the general constant, law, or equation F_u may re-generate interest in quantitative inspection of that old hypothesis.

In a similar sense, it will be noted in stable particles such as the dynamic Earth-moon body m_e–m_s that the relation $r_e\omega_e^2$ quantitatively defines the moon's gravity acceleration, $m_s r_e \omega_e^2$ its gravitational force, $m_s r_e^2 \omega_e^2$ its binding or gravitational potential energy, $r_e\omega_e$ its orbital velocity, $m_s r_e^2 \omega_e$ its orbital angular momentum, etc. The point of such elementary arguments is to suggest that formulas or equations such as $m_s r_e^3 \omega_e^2 (m_s m_e)^{-1} = r_e^3 \omega_e^2 m_e^{-1} = G$, $r_r B (m_e \omega_r)^{-1} = \mu_b$, or $m_s r_e^3 \omega_e^2 \omega_r^2 (r_e^2 B^2)^{-1} = F_u$ each embrace an almost infinite variety of definitions or statements of the universal laws governing particles and fields. Recognizing the many direct generalizations implied in the simple fact that $G\mu_b^{-2} = \hbar c L_m^{-2} = F_u = 3.981 \times 10^8$ dynes, it can then be appreciated that an almost infinite number of indirect correlations of matter and universe properties and actions are inherent in its brief and compact structure. At least, in its classical mechanical sense or use, it will be agreed that the equation of F_u expresses a surprising essence of both "reality" and simplicity.

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(1) A. T. GRESKY, "New Physical Constants from Dimensional Analysis," JOUR. FRANKLIN INST., Vol. 265, p. 85 (1958).

(2) A. T. GRESKY, "On Explanations of Electric and Magnetic Units and Constants," JOUR. FRANKLIN INST., Vol. 267, p. 201 (1959).

(3) A. T. GRESKY, "Universal Units of Mechanics, Magnetism, and Temperature," JOUR. FRANKLIN INST., Vol. 268, p. 388 (1959).

APPENDIX A

The data in Table I are provided for convenience of the reader in checking (numerically and dimensionally) the values and meanings of formulations (or laws) such as $r_{CD}{}^3\omega_{CD}{}^2 m_C{}^{-1} = G$, $r_{sC}B_C(m_C\omega_C)^{-1} = \mu_b$, $(\omega_{C_s}\omega_C\omega_{C_p}{}^{-2})^2 = \mu_b{}^2 G^{-1}$, $m_{ae}r_{ae}{}^2\omega_{ae} = \hbar$, $r_e B_e \omega_e{}^{-1} = L_m$, $r_e\omega_e = c$, $m_C r^3{}_{AC}\omega_{AC}$

TABLE I.—*Mass, Radii, Angular Velocities, and Magnetic Intensities of Macroscopic and Microscopic "Particles."*

(A) MACROSCOPIC (OR CLASSICAL MECHANICAL) PARTICLES

m_A = 197,000,000.0 $\times 10^{25}$ g = mass of Sun = 330,000 × Earth's mass
m_B = 33.5 $\times 10^{25}$ g = mass of Mercury = 0.055 Earth's mass
m_C = 598.0 $\times 10^{25}$ g = mass of Earth = 6.6 × 10^{22} tons = $r_{CD}^3\omega_{CD}^2/G$ (Conversion ratio = 9.072 × 10^6 g/short ton)
m_D = 7.36 $\times 10^{25}$ g = mass of Moon = 0.0123 Earth's mass

r_A = 695.0 $\times 10^8$ cm = equatorial radius of Sun = 430,000 miles × 1.61 × 10^5 cm/mile
r_B = 2.42 $\times 10^8$ cm = equatorial radius of Mercury = 1,550 miles × 1.61 × 10^5 cm/mile
r_C = 6.36 $\times 10^8$ cm = equatorial radius of Earth = 3,963 miles × 1.61 × 10^5 cm/mile
r_D = 1.735 × 10^8 cm = equatorial radius of Moon = 1,077 miles × 1.61 × 10^5 cm/mile

r_{sA} = 695.0 $\times 10^8$ cm = axial radius of Sun = 430,000 miles × 1.61 × 10^5 cm/mile
r_{sB} = 2.42 $\times 10^8$ cm = axial radius of Mercury = 1,550 miles × 1.61 × 10^5 cm/mile
r_{sC} = 6.36 $\times 10^8$ cm = axial radius of Earth = 3,950 miles × 1.61 × 10^5 cm/mile
r_{sD} = 1.735 × 10^8 cm = axial radius of Moon = 1,077 miles × 1.61 × 10^5 cm/mile

ω_A = 29.5 $\times 10^{-7}$ rad·sec^{-1} = angular velocity of Sun = 1.0 rev/25 days × 2π rad/rev × 1.157 × 10^{-5} day/sec
ω_B = 8.25 × 10^{-7} rad·sec^{-1} = angular velocity of Mercury = 1.0 rev/88 days × 2π rad/rev × 1.157 × 10^{-5} day/sec
ω_C = 729.0 × 10^{-7} rad·sec^{-1} = angular velocity of Earth = 1.0 rev/day × 2π rad/rev × 1.157 × 10^{-5} day/sec
ω_D = 26.6 × 10^{-7} rad·sec^{-1} = angular velocity of Moon = 1.0 rev/27.33 days × 2π rad/rev × 1.157 × 10^{-6} day/sec

B_A = 106,000.0 $\times 10^4$ sec^{-1} = 60.0 gauss = polar intensity of magnetization of Sun
B_B = 1.49 × 10^4 sec^{-1} = 0.00085 gauss = polar intensity of magnetization of Mercury = $\mu_b m_B\omega_B/r_{sB}$
B_C = 886.0 × 10^4 sec^{-1} = 0.51 gauss = polar intensity of magnetization of Earth
B_D = 1.47 × 10^4 sec^{-1} = 0.00084 gauss = polar intensity of magnetization of Moon = $\mu_b m_D\omega_D/r_{sD}$

r_{AB} = 580.0 $\times 10^{10}$ cm = radius of Mercury's orbit about the Sun = 36,000,000 miles × 1.61 × 10^5 cm/mile
r_{AC} = 1490.0 × 10^{10} cm = radius of Earth's orbit about the Sun = 93,000,000 miles × 1.61 × 10^5 cm/mile
r_{CD} = 3.84 × 10^{10} cm = radius of Moon's orbit about the Earth = 238,000 miles × 1.61 × 10^5 cm/mile

ω_{AB} = 8.26 × 10^{-7} rad·sec^{-1} = angular velocity of Mercury's orbit about the Sun = 1.0 rev/0.24 year × 2π rad/rev × 3.17 × 10^{-8} year/second
ω_{AC} = 2.0 × 10^{-7} rad·sec^{-1} = angular velocity of Earth's orbit about the Sun = 1.0 rev/year × 2π rad/rev × 3.17 × 10^{-8} year/second
ω_{CD} = 26.6 × 10^{-7} rad·sec^{-1} = angular velocity of Moon's orbit about the Earth = 1.0 rev/27.33 days × 2π rad/rev × 1.157 × 10^{-5} day/sec

(ω_{C_p} = 7.65 × 10^{-12} rad·sec^{-1} = angular velocity of precession of the Earth = 1.0 rev/26,000 years × 2π rad/rev × 3.17 × 10^{-8} year/second)
(ω_{C_n} = 10,700.0 × 10^{-12} rad·sec^{-1} = angular velocity of nutation of the Earth = 1.0 rev/18.6 years × 2π rad/rev × 3.17 × 10^{-8} year/second)

TABLE I.—*Continued.*

(B) MICROSCOPIC (OR QUANTUM-MECHANICAL) PARTICLES

$m_a = 167{,}000{,}000.0 \times 10^{-32}$ g = mass of neutron (or proton) = atomic mass unit/6.02×10^{23} (Avogadro's number)
$m_b = 91{,}100.0 \times 10^{-32}$ g = mass of positron (near hydrogen nucleus) = $m_a/1840$
$m_e = 91{,}100.0 \times 10^{-32}$ g = mass of electron (in Bohr orbit about proton) = $m_a/1840$
$m_d = 2.41 \times 10^{-32}$ g = mass of photon (associated with 27.12 ev electron = 13.56 ev/c^2

$r_a = 18.4 \times 10^{-14}$ cm = equatorial radius of the neutron = $ca(\hbar^2/m_a \times 11.6 \text{ Mev})^{\frac{1}{2}}$, (Mev = 1.602×10^{-6} g cm^2 sec^{-2})
$r_b = 3{,}860.0 \times 10^{-14}$ cm = equatorial radius of the positron = $(\hbar^2/m_b \times 0.511 \text{ Mev})^{\frac{1}{2}}$
$r_e = 3{,}860.0 \times 10^{-14}$ cm = equatorial radius of the electron = $(\hbar^2/m_e \times 0.511 \text{ Mev})^{\frac{1}{2}}$
$r_d = 145{,}000{,}000.0 \times 10^{-14}$ cm = equatorial radius of the photon = $\hbar c/13.56$ ev

$r_{za} = 9.75 \times 10^{-14}$ cm = axial radius of the neutron = $\hbar/2m_a r_a \omega_a = 3.32 \times 10^{-39}$ cm$^3/r_a^2$
$r_{zb} = 1{,}930.0 \times 10^{-14}$ cm = axial radius of the positron = $\hbar/2m_b c$
$r_{ze} = 1{,}930.0 \times 10^{-14}$ cm = axial radius of the electron = $\hbar/2m_e c$
$r_{zd} = 145{,}000{,}000.0 \times 10^{-14}$ cm = axial radius of the photon = $\hbar c/13.56$ ev

$\omega_a = 1{,}760{,}000.0 \times 10^{16}$ rad·sec^{-1} = angular velocity of neutron = 11.6 Mev/$\hbar$
$\omega_b = 77{,}700.0 \times 10^{16}$ rad·sec^{-1} = angular velocity of positron = 0.511 Mev/$\hbar$
$\omega_c = 77{,}700.0 \times 10^{16}$ rad·sec^{-1} = angular velocity of electron = 0.511 Mev/$\hbar$
$\omega_d = 2.07 \times 10^{16}$ rad·sec^{-1} = angular velocity of photon = 13.56 ev/$\hbar$, (ev = 1.602×10^{-12} g cm^2 sec^{-2})

$B_a = 514{,}000{,}000.0 \times 10^{14}$ sec$^{-1} = 2.92 \times 10^{18}$ gauss = intensity of magnetization of neutron = 9.76×10^{-24} cm^3 gauss/3.32×10^{-39} cm^3
$B_b = 56{,}700.0 \times 10^{14}$ sec$^{-1} = 3.23 \times 10^{11}$ gauss = intensity of magnetization of positron = 9.273×10^{-21} cm^3 gauss/$r_b^2 r_{zb}$
$B_e = 56{,}700.0 \times 10^{14}$ sec$^{-1} = 3.23 \times 10^{11}$ gauss = intensity of magnetization of electron = 9.273×10^{-21} cm^3 gauss/$r_e^2 r_{ze}$
$B_d = 0.00004 \times 10^{14}$ sec$^{-1} = 2.28 \times 10^{2}$ gauss = intensity of magnetization of photon = $L_m \omega_d / r_{zd}$

$r_{ab} = 3.86 \times 10^{-11}$ cm = radius of 0.511 positron's orbit about the neutron
$r_{ae} = 530.0 \times 10^{-11}$ cm = radius of 27.12 ev electron about the proton (neut + positron)
$r_{ed} = 145{,}000.0 \times 10^{-11}$ cm = radius of 13.56 ev photon about the electron (or hydrogen atom)

$\omega_{ab} = 77{,}700.0 \times 10^{16}$ rad·sec^{-1} = orbital angular velocity of the positron about the neutron
$\omega_{ae} = 4.14 \times 10^{16}$ rad·sec^{-1} = orbital angular velocity of the electron about the proton (neut + positron)
$\omega_{ed} = 2.07 \times 10^{16}$ rad·sec^{-1} = orbital angular velocity of the photon about the electron (or hydrogen atom)

$(r_{eA}B_A r_{eC}B_C)^{-1} = F_u$, $m_e r_{ee}^2 \omega_{ee}\omega_e\omega_e^2(r_{ee}B_e B_e)^{-1} = F_u$, etc. Such checking would be considered vital to the perception and understanding of the constant or law of F_u, as well as its ultimate simplicity in the correlations of "real" measurements, particularly in the case of astronomic bodies.

It will be noted that the four microscopic particles may be considered to compose the essential components of a loosely-constructed single particle, the hydrogen atom, that is, if one perceived it in terms of a purely classical model. As such, the neutron (containing nearly all of the mass) may be likened to the macroscopic body, the Sun; the positron (which contributes the "positive charge" to the hydrogen nucleus) may similarly be loosely compared to Mercury, the electron to the Earth, the photon to the Moon, etc., so that the two sets of data may imaginatively compare two specific "particles," one in the vastness of "outer space" and another in the minuteness of "inner space."

In the case of the neutron, it may be noted that $m_n/1.333\pi r_n^3 r_{en}$ and $r_n^2 r_{en} B_n$ describe an oblate spinning body with a density restricted to 1.2×10^{14} g cm^{-3} (atomic nuclear density) and a magnetic moment of 9.65×10^{-24} cm^3 gauss (or 1.91 nuclear magnetons). It will be noted also to have a spin $m_n r_n r_{en}\omega_n$ of 5.27×10^{-28} g cm^2 sec^{-1} (or $\hbar/2$), a fine structure or coupling constant $B_n\omega_n^{-1}$ of 2.92, and a quadrupole moment $r_n^2 r_{en} B_n(r_{en}\omega_n)^{-1} = 5.66 \times 10^{-27}$ cm^3. In general, the concept that energetic microscopic particles of 13.56 ev to 11.6 Mev exhibit rotary or orbital angular velocities in the order of 10^{16} to 10^{22} radians·sec^{-1} is perhaps as difficult for the mind to imagine as the smallness of the particles' volumes, for example, 10^{-30} cm^3 to 10^{-18} cm^3. Though such fantastic angular velocities are certainly not measurable *per se*, it will be noted that they differ by only a factor of 2π from the frequencies (waves·sec^{-1} or revolutions·sec^{-1}) granted to be associated with particles of such energies. Magnetic intensities of 10^3 to 10^{18} gauss (up to almost 10^{16} times as intense as the Earth's field are also difficult to imagine; however, it will be realized that such intensities characterize areas as inconceivably small as 10^{-36} cm^3.

The illustration that F_u may accurately describe particles (or prescribe their related motions and magnetizations) with areas as small as 10^{-36} cm^3 or as large as 10^{36} cm^3, or with masses as small as 10^{-32} g or as great as 10^{32} g, is an intriguing possibility which the data in Table I attempt to convey. The additional possibility that F_u is a valid "law" for masses operating in restricted space (for example, constant density) as well as in unrestricted- or free-space (for example, the perfect vacuum), is perhaps an example of the uniqueness which would hopefully be inherent in a general law of classical and quantum mechanics.

APPENDIX B

Quantum mechanics had its beginnings in attempts to explain the complicated continuum of light wave-lengths emitted by excited hydrogen atoms. On the basis of classical models, the planetary electron was initially considered to move around the proton or hydrogen nucleus in a primary circular orbit of about 5.3×10^{-9} cm radius, and was capable of transference to and from secondary orbits which were discretely and harmonically related to the primary radius.

Spectral evidence suggested that excitation of the hydrogen electron in its normal or ground state (13.56 ev) could cause it to jump to higher energy levels (or shorter orbital radii) such as quantum state 2 (16.97 ev), state 3 (15.07 ev), state 4 (14.41 ev), etc.; and, on return to the ground state, it could emit the excess energy in the form of continuous electromagnetic radiation, for example, a total of 3.41 ev from the second state, 1.51 ev from the third state, and 0.85 ev from the fourth state, etc. Thus originated the mathematical and wave-mechanical analyses of the Lyman, Balmer, Paschen, Brackett, and other series, and their continuum of characteristic light wave-lengths. Various forms of experimental evidence have amply verified many predictions based on such quantum mechanical interpretations. However, it may be interesting to speculate, for purely academic reasons, on another interpretation which might have occurred to the early theoreticians. For instance, it can be recognized that the effective or deBroglie wave-lengths ($\lambda = \hbar/mv$) or radii ($r = \hbar/mv$) of the 13.56, 3.41, 1.51, and 0.85 ev electrons are integrally related. As shown in Table II, the four underscored radii exemplify

TABLE II.—*Integral Relationships of "Hydrogen Electron Radii."*

	Photon Wave-length		Electron Radii	
	λ, cm·wave^{-1}	$(E_k)_n$, ev**	r_r, cm·rad^{-1}	E_p, ev**
Lyman Series				
Limit	0.91×10^{-5}	13.56	0.53×10^{-8}*	27.12
H_α line	1.22×10^{-5}	10.2	0.61×10^{-8}	20.4
Balmer Series				
Limit	3.65×10^{-5}	3.41	1.06×10^{-8}	6.81
H_β	4.861×10^{-5}	2.58	1.22×10^{-8}	5.16
Paschen Series				
Limit	8.25×10^{-5}	1.51	1.59×10^{-8}	3.02
H_γ line	10.94×10^{-5}	1.13	1.83×10^{-8}	2.26
Brackett Series				
Limit	14.6×10^{-5}	0.85	2.12×10^{-8}	1.70
H_Δ line	19.4×10^{-5}	0.63	2.43×10^{-8}	1.26

* First Bohr radius.
** ev = 1.6×10^{-12} g cm^2 sec^{-2}; $E_p = 2E_k$; $\lambda = hcE_k^{-1}$; $r_r = (h^2 m_e^{-1} E_p^{-1})^{\frac{1}{2}}$.

such integral relationships. These radii would be characteristic of electrons with potential orbital energies E_p of 27.12, 6.81, 3.02, and 1.7 ev, which may be considered to have associated photons (of energy = $E_p/2$) with emission wave-lengths of 0.91, 3.65, 8.25, and 14.6×10^{-5} cm, normally observed as the spectral series limits. Whereas the electron radii increase in length by factors of 2, 3, 4, etc., the associated wave-length limits are observed to increase by factors of $(2)^2$, $(3)^2$, $(4)^2$, etc. Similar factors of difference are also associated with relationships between the E_p's and E_k's.

As shown in Table III, it might have seemed that the electron could conceivably occupy

TABLE III.—*Energy Limits or the Hydrogen Spectral Series.*

	Photon Wave-length λ, cm·wave^{-1} $\times 10^5$	Electron Radius r_r, cm·rad^{-1} $\times 10^8$	Kinetic Energy E_k, ev	
Lyman	0.911	0.529	13.56	(E_1)
Balmer	3.65	1.06	3.41	(E_2)
Paschen	8.2	1.59	1.51	(E_3)
Brackett	14.6	2.12	0.85	(E_4)
Pfund	22.8	2.65	0.54	(E_5)
Humphreys	32.8	3.18	0.38	(E_6)
—	44.6	3.71	0.28	(E_7)
—	—	—	—	—

alternate states or orbital positions at radii up to greater than 31.8×10^{-9} cm (or some six times the radius of the primary orbit), and be capable of interactions (absorption or emission) with photons of resonant frequencies. However, such an interpretation would apparently have no logical physical basis to explain energy conservation.

An alternate academic point which seems worth making, in a discussion which centers on a possible generalization of classical and quantum physics, is the suggestion that perhaps a more causal explanation of these hydrogen electron orbits is possible. For instance, the spectral evidence of electrons with discrete energies E_3, E_5, E_6, etc., may be thought to reflect the presence, during the experiment of excitation, of non-planetary or bombarding electrons which collide with the 27.12 ev hydrogen electron by virtue of their probability for specific resonance (owing to their integrally related radii) with its 0.53×10^{-8} cm orbital "lattice." Such electron collisions, in turn, might be considered to cause transference or emission of their kinetic energy in the form of the observed photons, and also to cause a minor outward orbital displacement

of the "slowed down" planetary electron. The data in Table IV, showing the light wavelengths and electron radii characteristic of the Lyman series, suggest that such a displacement might be a maximum of about 0.08×10^{-8} cm ($0.61 - 0.529 \times 10^{-8}$ cm), that is, caused by

TABLE IV.—*Energy Levels of the Lyman Series: Hydrogen Spectra.*

Photon Wave-length λ, cm·wave$^{-1} \times 10^8$	Electron Radius r, cm·rad$^{-1} \times 10^8$	Kinetic Energy E_k, ev	
0.911	0.529	13.56	(E_1)
—	—	—	—
0.933	0.535	13.28	(E_1–E_7)
0.94	0.537	13.17	(E_1–E_6)
0.95	0.541	13.00	(E_1–E_5)
0.972	0.548	12.7	(E_1–E_4)
1.026	0.562	12.05	(E_1–E_3)
1.216	0.61	10.15	(E_1–E_2)

the planetary electron's loss of 6.82 ev of its orbital potential energy (or 3.41 ev of its kinetic energy). It will be noted that each of the other Lyman lines (and/or the corresponding electron radii) may reflect similarly discrete losses of E_3, E_4, E_5, or E_6, etc., that is, to specific non-planetary electrons having radii integrally related to the planetary orbit. The additional temporary or experimentally produced orbits at 0.61, 0.562, 0.548, 0.541×10^{-8} cm, etc., might then be considered to establish a multiple lattice structure (in a hydrogen sample) capable of separate resonance interactions with another group of non-planetary electrons, for example, those associated with the H_α, H_β, H_γ, and H_Δ lines in Table II. The persistent lines shown in Table V might then seem to reflect that the highest probability of resonance exists for collisions with non-planetary electrons having the specific E_2, E_3, and E_4 energies.

TABLE V.—*Persistent Lines of Hydrogen Spectrum.*

Photon Wave-length λ, cm·wave$^{-1} \times 10^8$	Electron Radius r, cm·rad$^{-1} \times 10^8$	Kinetic Energy E_k, ev	
1.216	0.61	10.15	(E_1–E_2)
4.861	1.22	2.56	(E_2–E_4)
6.563	1.42	1.9	(E_2–E_3)**

** $(E_1$–$E_3) - (E_1$–$E_2)$.

In summary, it may be suggested that the complex spectra are reflections of experimental conditions of excitation rather than an apparent randomness of the planetary electron's normal orbital position or energy. Sharp interactions with electrons of 13.56 ev kinetic energy, that is, the ionization potential of the hydrogen electron, would imply that the exact equivalence of radii would play a quite important part in the probability of collision.

Though such unsophisticated "models" have a highly unpopular flavor in modern theory, it may seem worthwhile to ask occasionally whether causality or "reality" cannot play a significant (if not predominant) role in the affairs of the submicroscopic particles, as well as in the systems of classical mechanics. If indeed a general law of physics is ever established, such philosophical questions of causality vs. indeterminism may be resolved or compromised to some extent, perhaps to the benefit of our admittedly frail human perception of natural forces.

THE FRANKLIN INSTITUTE

PROCEEDINGS OF THE STATED MEETING HELD NOVEMBER 18, 1959

Presentation of the Vermilye Medal and Observance of the 135th Anniversary of The Franklin Institute

RECEPTION AND DINNER

On Wednesday, November 18, 1959, a large crowd of Institute members and guests gathered to observe the 135th Anniversary of the founding of the Institute and to witness the presentation of the 1959 Vermilye Medal to George Romney, President of American Motors Corporation. A reception honoring Mr. Romney was held at 6:30 P.M. in the lobby of Franklin Hall, and at 7:05 P.M. the guests assembled for dinner. The Reverend William X Smith, Rector of St. Mary's Church, Hamilton Village, asked the blessing.

During dinner, Wynn Laurence LePage, President of the Institute, proposed the following toast to Benjamin Franklin:

> "Let us arise in salute to that most able of men, who, by carefully planning his business affairs, was able to retire from active business life at the early age of 42—younger by some years than many of us here! To Benjamin Franklin!"

Mr. LePage also noted the fact that six other Philadelphia organizations celebrated their 135th anniversaries in 1959: St. Mary's Church, the Historical Society of Pennsylvania, Jacob Reed's Sons, John Sidebotham, Inc., The American Baptist Publication Society and The American Sunday School Union.

STATED MEETING

Mr. LePage opened the Stated Meeting at 8:15 P.M. The minutes of the Stated Meeting held October 21 were approved as published in the November issue of the JOURNAL. On motion of Richard T. Nalle, seconded by James H. Robins, the proposed amendments to Article IV, Section 5 and Article VIII, Section 3 of the By-Laws were adopted. Mr. LePage then briefly reviewed the 135-year history of the Institute, in the following words:

HISTORY OF THE FRANKLIN INSTITUTE

BY

WYNN LAURENCE LePAGE

Mr. Romney, Honored Guests, Ladies and Gentlemen.

Outstanding and significant facts and events underlie our purposes this evening. We celebrate the One Hundred Thirty-Fifth Anniversary of The Franklin Institute, the Twenty-Fifth Anniversary of this fine edifice on the Benjamin Franklin Parkway in Philadelphia, and of this great Memorial Hall, and, by no means lastly, we are here to give signal recognition for achievement in industrial management to an American businessman, and a good friend.

One hundred and thirty-five years ago, in 1824, James Monroe was finishing two terms in the White House and the famous Monroe Doctrine was but a year old. It was an election year and, strange as it may seem today, there were no party issues at stake at all. Four men ran for the office of President, all of them Republicans! In that year, it was the "man" who had the following that counted, not the Party. The men who ran were John Quincy Adams, Henry Clay, William H. Crawford and Andrew Jackson. Mr. Adams, you will remember, won the election and became President of the United States the following year.

At that same time, there were other things going on. There were Seminole Indian raids in Florida and Georgia, the Erie Canal was almost finished, Mexico became a republic, the first railway ran in England and Charles the Tenth was on the throne in France. In case anyone is interested, King George the Fourth reigned in Great Britain. But to narrow our viewpoint down to Philadelphia, the outlook was something like this.

Mayor Joseph Watson was having some problems with street paving, with City taxes and with plans for the imminent visit of the Marquis de Lafayette. Lafayette arrived on time. There were parades and fetes and long speeches saluting the return of a comrade-in-arms of some fifty years earlier and I am told that everything went off very well!

And there was another event in Philadelphia in the early days of 1824 that bears particular noting this evening. A twenty-two-year-old Maine Yankee, one Samuel Vaughan Merrick, found himself the owner of a small machine shop. Unfortunately, Merrick was without the slightest vestige of mechanical training. But, having great determination and initiative he set out to improve himself by attempting to join an association of mechanics. His request for membership was rejected! I don't think we would be here this evening if he had been accepted. Thoroughly provoked, he called a public meeting to organize his own society. The meeting was a complete and utter failure! Twice burned,

he sought out another young man, a Dr. William Keating, then a teacher in Chemistry at the University of Pennsylvania.

The two of them, young and inexperienced, surrounded themselves with several other young and unknown men and, scanning the City Directory, selected about 1500 names, to each of whom they sent a personal invitation to attend an organization meeting of a proposed new society for the lay study of mechanical science.

This time, Merrick and his associates met with success. The meeting room in the Old County Court House adjoining Independence Hall overflowed with interested people and, then and there, The Franklin Institute was conceived. The membership within the first year numbered over 500 and ran the gamut of occupations from a president of the United States Bank to a saddlemaker and to a piano tuner who incidentally was named Thomas Loud!

On February 14, 1824, there appeared in the *Evening Post*, the following news item, and I should like to quote it to you.

> A Society, under the title of The Franklin Institute, has just been established in this city, which promises to be of considerable usefulness. Its object is to advance the general interest of Manufactures and Mechanics, by extending a knowledge of mechanical sciences to its members, and others, at a cheap rate. They propose, as the best means of effecting this object, the establishment of popular lectures, by the formation of a cabinet of models and minerals, and of a library, and by offering premiums on all useful improvements in the mechanic arts. We believe three dollars is to be the yearly expense of membership, and twenty-five dollars to constitute a membership for life.
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> Nicolas Biddle, Esquire (a member of The Franklin Institute and President of the United States Bank), in a letter on this subject, has the following observations:
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> "I know of no enterprise which promises more general utility than the effort to connect the theory with the exercise of these arts and to blend science with practical skill. There are few mechanical operations which do not essentially depend on fixed principles, the knowledge of which cannot fail to increase the dexterity of all those who are engaged in them, while the habits of enquiry, of study and reflection, which the pursuit of this knowledge requires, diffuse over their general character and manners a spirit of intelligence which improves at once the work and the workman."

Now, let us see just what has happened to The Franklin Institute to support the prediction of its promising to "be of considerable usefulness."

One of the first actions of the Board of Managers of the early Franklin Institute was to form a library, and a Library Committee, the first committee of The Franklin Institute. The Library Committee exists today and, of course, The Franklin Institute Library in Pepper Hall is one of the finest technical libraries in the world.

Then, with an eye to the importance of high quality of workmanship in the manufacture of products, and to encourage original work in the mechanic arts, the Board set up a committee which exists today as the Committee on Science and the Arts. Without a break for 135 years, this Committee has functioned and now recommends for award today's formidable array of medals of The Franklin Institute. There is, I believe, no other group in the world resembling this able, objective and superbly qualified group of men who continuously comb the fields of the physical sciences and technological accomplishment and recommend for award the most deserving scientists and engineers the world over. It is largely the work of this Committee that has established the awards of The Franklin Institute among the greatest sought after.

Another action of the first Board of Managers of The Franklin Institute, in 1824, was the appointment of a Committee on Instruction. This Committee created the lecture series called for by the founders and in those days, professorships were established in chemistry, natural philosophy, mechanics and architecture, and capable instructors were procured. There were, in addition, volunteer lecturers from the membership of the Institute. Thereafter a regular formal school of mechanical and architectual drawing was opened and was so successful that other courses were opened in English literature and even—believe it or not—in ancient and modern languages.

I am pleased to recount that in 1827, over three hundred students were on the roll and it was the success of this new educational venture that led to the modeling of Philadelphia's Central High School and created the pattern for the secondary schools of our public school system today.

With the opening of the first public high school, Central, the Institute abandoned its own school as no longer necessary. The drawing school, however, continued well into the twentieth century.

That records of the activities of the new organization should be properly kept in order to provide a means for the publication of treatises and papers by the members, the JOURNAL OF THE FRANKLIN INSTITUTE was founded in 1826, its first issue appearing in January of that year. The JOURNAL is the second oldest such publication in America and today still serves its original purposes. It is highly esteemed in the scientific world and may be found on the shelves of the great libraries the world over.

At the time of the founding of the Institute, the so-called Industrial Revolution was making itself felt throughout the civilized world, in America, and most especially in Philadelphia, port city to the wealth of the Middle Atlantic States. This great change in the economic, scientific, technological and social structure of our civilization greatly affected The Franklin Institute—and was greatly affected by it. The Industrial Revolution was made possible by men of imagination, courage and of

great faith in God, in America, and in themselves. Men of this calibre were the founders and members of The Franklin Institute, and they are its members today.

At your places on the dinner table this evening, you have found a chronology of The Franklin Institute. I hope you will read it, keep it and refer to it from time to time. It might just be that The Franklin Institute is currently adding one more paragraph to that chronology.

One of the important perquisites of the Institutes at its founding was the so-called "cabinet of models." Today this "cabinet" has become the great Science Museum which surrounds us in this building and which yearly attracts hundreds of thousands of children and adults of all ages who come to see how mechanical things work and how scientific phenomena are explained.

But the Science Museum and this great Memorial did not spring into being overnight. In the middle of the 1920's, the Board of Managers of The Franklin Institute and many of its members realized that the Institute had outgrown its 100-year old building on Seventh Street in Philadelphia. Accordingly, building plans were carefully made, plans that would include a Science Museum based on the exhibit techniques of the Deutches Museum in Munich. At the same time, the Poor Richard Club, the oldest advertising club in the world, also was planning a fitting memorial for its namesake, there being no national memorial to this great man, Benjamin Franklin. In true Philadelphia fashion, The Franklin Institute and the Poor Richard Club put their plans together.

That proper recognition may be made, I shall ask a member of the Board of Managers of The Franklin Institute briefly to stand. Mr. Morton Gibbons-Neff. Mr. Neff was president of the Poor Richard Club in the mid '20's and he it was who supplied much of the drive, the push, the energy in our cooperative striving toward this goal.

This evening, too, in recognition of the continued close cooperation which exists between the Poor Richard Club and The Franklin Institute, I turn to greet Mr. A. Edward Morgan, 1st Vice President of the Poor Richard Club.

In June, 1930, at the Benjamin Franklin Hotel, a group of over 2000 civic leaders met and in the subscription drive which followed that meeting, $5,100,000 was raised in 12 days to swell the fund for a truly great memorial to America's first practical scientist, a businessman and a statesman. This great edifice, the Science Museum and Memorial Hall, permeated with the spirit of him in whose honor it stands, is a truly fitting memorial to Benjamin Franklin.

In November of 1933, the first part of the building was opened to the public, the Fels Planetarium, the instrument of which was the gift of the late Samuel S. Fels.

Two months later, on January 1, 1934, the Science Museum and

Memorial were opened. So, you see that this is a double anniversary for us, the 135th anniversary of The Franklin Institute, and the 25th anniversary of this Museum and great Memorial.

In May 1938, we dedicated these facilities in the presence of educational and civic leaders of the nation and the world. We dedicated, too, this Memorial Hall, and the heroic statue by James Earle Fraser, given by William McLean.

In the great Science Museum around us, each year we welcome hundreds of thousands of adults and children—this year, well over 400,000 of them. The students come to augment their knowledge, and to use the Museum as an extension of their classrooms. The adults come here to be brought up-to-date on the scientific and technological achievements of the times, and to keep up with the active and inquiring minds of their children. From the smell of crude oil in the boring from an oil well to the sound of an unintelligible radio signal from the depths of the Milky Way, the Science Museum exhibits help us all understand the world and the universe.

You will find listed in that booklet on your tables some of the scientific research activities of the committees of The Franklin Institute conducted in the early years. Today, we have The Franklin Institute Laboratories for Research and Development, now a major activity of the Institute. Formally established in 1946, the Research and Development Laboratories, using the same or similar teamwork techniques of the early years, have performed and are performing a tremendous service for Government and industry. But that is a story in itself, a story too long for the telling this evening.

There are other research stories here too, for example, the basic nuclear research carried on by the Bartol Research Foundation of The Franklin Institute, in its laboratories at Swarthmore. For years under the direction of that internationally well-known physicist, Dr. W. F. G. Swann, the Bartol staff are now guided by his protege and colleague, Dr. Martin A. Pomerantz.

And in Delaware, at Newark, the Biochemical Research Foundation, of which The Franklin Institute is the trustee, delves into the mysteries of cell growth, both normal and abnormal. Dr. William G. Batt is the Director.

This is the 135th Anniversary of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts. Perhaps we *have* been and *are* of "considerable usefulness," as that article in the February 14, 1824, issue of the *Evening Post* predicted. We will most certainly continue to live up to the charge of our history. And in so doing we shall draw strength from the warm friends who surround us, the business, cultural, and educational community, and the millions of people who have come to depend upon The Franklin Institute as their source of intellectual inspiration in matters scientific.

PRESENTATION OF THE VERMILYE MEDAL

MR. LEPAGE: "I now call upon Mr. Francis J. Chesterman, a member of the Board of Managers of The Franklin Institute and Chairman of the Vermilye Medal Committee of The Franklin Institute."

MR. CHESTERMAN: "The Vermilye Medal, which will be presented this evening, is the result of a gift of a sum of money by William M. Vermilye on December 30, 1936 for the purpose of (1) providing a permanent trust fund in the custody of The Franklin Institute to establish and support a Vermilye Medal, and (2) this medal to be awarded biennally by The Franklin Institute 'in recognition of outstanding contribution in the field of industrial management.' We remember Mr. Vermilye as a member of The Franklin Institute from 1936 until his death on August 29, 1944. During part of this time he was a member of the Board of Managers of The Franklin Institute.

"The Vermilye Medal has been presented to seven individuals: 1938, to Lewis H. Brown; 1941, to William S. Knudson; 1943, to Walter S. Gifford; 1946, to Martin W. Clement; 1950, to J. Howard Pew; 1953, to Benjamin F. Fairless; and 1957, to Frederick C. Crawford.

"Perhaps the best way to describe the man we honor tonight is that George Romney stands out as a purposeful man who knows exactly where he is going. His accomplishments—which have been described as spectactular--certainly tend to prove that statement. His story has been well recorded in important publications. He has been interviewed on a trans-Atlantic broadcast and on televison programs. In each case it has been clear that George Romney knows what he is doing and where he wants to go. Here is a man dedicated to certain principles and ideas that are readily understandable. It is important that we realize that George Romney is a man of deep convictions—and they do not merely include a fervent belief in the smaller compact car. He has supported, by discussion and in other ways, a return to forceful debate and wider communication of critical problems. He believes in the American principle of unbridled, free competition, and of keeping political and economic power apart.

"He is a deeply religious man—one of the nation's outstanding Mormons—a man happy with his family, and one who loves vigorous exercise and vigorous play.

"He has demonstrated that he is a man of tremendous initiative, as is evidenced by his record for predicting the future. He predicted the popularity of the small car. He predicted the single-unit body construction pioneered by his company in 1940. In 1957, following three years of heavy losses when the experts and the columnists, and seemingly everybody, were writing epitaphs for a struggling company's demise, Mr. Romney predicted that American Motors would be in the black before the end of the year—and it was. Since then, payment of

dividends has been restored, the company has doubled its employment and is now the largest taxpayer in the State of Wisconsin.

"Another of Mr. Romney's predictions was that foreign small cars and compact cars would account for 8 per cent of the total United States sales in 1958. This was looked upon as a very wild guess. Actually, the result was 11 per cent of total sales. So far in 1959 nearly 16 per cent of the cars sold have been smaller cars.

"Mr. Romney goes further and predicts a much further increase in the use of smaller cars—perhaps up to 60 per cent—and that his company will be getting a substantial share of that market. He has been called the 'Apostle of the Small Car,' and 'Man Who Was Right' and the 'Crusader for the Compact Car.'

"Mr. President, it is a pleasure and an honor to present Mr. George Romney, President of the American Motors Corporation, to receive the Vermilye Medal, for—and I quote the citation—

> For forthright courage and intuitive vision in the development and manufacture of a competive product and in overcoming great difficulties.
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> To this man, dedicated to certain principles and ideas which he has developed consistently through his positive leadership and within an aggressive organization, an acknowledged authority on employee relations both national and international, and a distinguished exemplar of the American way of life and the free enterprise system this award is presented.

MR. LEPAGE: "Mr. Romney, as President of The Franklin Institute, I present to you this Vermilye Medal for 1959 and the certificate which accompanies it.

"With it, you have my most warm and sincere congratulations."

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Mr. Romney accepted the medal and delivered the address which appears in full on the following pages.

At the close of the address, Mr. LePage thanked Mr. Romney and adjourned the meeting at 9:45 P.M.

ARE WE CONFUSED ABOUT AMERICA'S DIRECTION? *

BY

GEORGE ROMNEY [1]

Thank you very much. I am humbly grateful for this distinguished award. Few men are so honored without protesting their lack of adequate merit. But protests themselves can be immodest—if we believe, as I do, that this kind of generous recognition is intended to go beyond individuals, reaching through them to honor high principles. So, with my sincere appreciation, permit me to join you tonight in honoring the great purposes of this institution.

I hope you will consider it consistent with these purposes for me to speak earnestly on a truth that I believe has a profound bearing on the future of this nation and of the world.

Someone is usually heard proclaiming impending disaster. This is not my intent. On the contrary, I am caught in a sense of excitement over what our achievements so far promise for tomorrow, and how near we may be to the breakthrough in human affairs for which I believe this nation was divinely created.

But first, we have some impediments that need clearing away. Our central target today, in my view, should be the excessive power that is growing like great clots in the nation's arteries.

For our heritage, we are strangely blasé about excessive power.

When our constitution was being drafted here, the framers seemed as sensitive to freedom—and its antithesis—power—as if each had just made an escape from bondage. You cannot read the accounts of that momentous spring and summer here without feeling that the air was charged with the ardor of liberty. And you cannot consider liberty without considering power.

No matter where you place your finger on the debates—whether the relative power of the states *versus* the federal government; the large states *versus* the small states; Southern slavery or Northern commerce; suffrage for freeholders only; the Senate's power *versus* the House's power; the Executive's power, the Supreme Court's power; aristocracy *versus* democracy—no matter where you listen in on the arguments (with Dr. Franklin striving throughout, counselling and seeking unanimity), the balancing and dispersion of power are at the core.

In his fascinating new book "Advise and Consent," Allen Drury, the author, has Bob Munson, the Senate Leader, say this:

* Delivered at the November 18, 1959 Stated Meeting of the Institute, in acceptance of the Vermilye Medal.

[1] President, American Motors Corporation, Detroit, Mich.

> The whole story of the operation of the American government is the story of the deliberate diffusion of power; the whole story of the life of the American government is the story between the deliberate diffusion of power, which in and of itself is perhaps the most brilliant device for the protection of liberty ever conceived by free men, and the deliberate concentration of power which free men have often deemed necessary to preserve liberty. If either diffusion or concentration were to win the battle once and for all then I think freedom would finally be gone and the great dream of liberty within discipline which is America would be gone forever.

We face the early prospect of concentration winning once and for all, and seem little concerned.

How is it that our forebears were so sensitive to power, and we have grown so forgetful?

Here are some of the reasons:

1. The material abundance produced by freedom has caused us to confuse bigness and power. Bigness and its benefits are cited as justification of what has become excess power.

2. There is the theory of countervailing power, one big power group offsetting another. This "balance of power" theory has always failed, and the balancing of excess industry power with excess union power is now threatening us with permanent concentration of excess Government power.

3. The complexity and magnitude of our present problems create individual confusion, uncertainty, and irresponsibility. Because all progress starts with the capacity of individuals this must be corrected through citizens informing themselves and exercising their responsibilities, or concentration will win by default.

4. Indifference and apathy resulting from our extraordinary abundance and superficial security. Most civilizations have expired on the bed of luxury. America has the opportunity to create a great new age by resisting this historical temptation.

Is anything plainer in history than the fact that concentrated power, whether it is "countervailing" or not, always finds its final expression in domination?

We are face to face with the dangers of such domination.

Excessive power of industry required reforms that reduced that power and opened the door to greater power for unions. Our laws became two-faced, banning monopoly power in the market place, but inducing it in collective bargaining. The steel strike is a forceful illustration of the result. The powerful unions banded together. The steel industry massed itself. And there they stood, two massive antagonists, both far greater in their strength than some foes whose struggles changed the course of civilization.

A point and a question are clear today.

First, the question: Must we wait for a whole series of national emergencies such as the steel strike to throttle us before acting—isn't it far wiser to tackle the causes of the national steel emergency before others arise?

The discouraging thing is that no one of consequence in the national political leadership is discussing how we prevent such national emergencies, but only what do we do when one occurs.

The point: Power begets power. If it breeds excessively in one place, it encourages it—even demands it—in another. Labor and industry strive to outdo each other in their strength, and their impasse can only result in the necessity for still greater power in a third quarter—in government.

Let me discuss these three power areas separately.

Powerful economic interests have always undertaken to dominate the country.

The tendency was resisted by our strong presidents.

The first big antitrust crusade of Presidents Theodore Roosevelt, Taft and Wilson was followed by the passage of the Clayton Act, prohibiting specific monopoly practices. This act, however, excluded labor organizations from the antitrust laws, which was the genesis of immunity for Labor in the exercise of monopolistic powers and abuses. It is this exclusion that provides the major point of conflict in the nation's economic policy. That policy regulates industrial activity on the basis of competition and promotes collective bargaining on the basis of monopoly.

Not until the Government began promoting the expansion of union power as an offset to business power did collective bargaining exemption from the competitive principle become an economic weapon of blockbuster proportions.

After passage of the Wagner Act in 1935, union power was substituted for business and agricultural power as the most important single political influence in our country. Today, we are faced with inherent dangers of excessive union power as well as excessive industry power.

The one sound and adequate solution is the elimination of the conflict in our national economic policy and adequate division of collective bargaining power. This country cannot progress economically half competitive and half monopolistic. Such economic conflict is as destructive of economic progress as our earlier division over slavery and freedom was to our social progress.

Are there alternatives to a change in national collective bargaining policy? Some people say all we need or should first attain is impartial enforcement of present local, state and federal laws and sound judicial interpretation of present laws. Certainly there are flagrant instances of failure impartially to enforce local ordinances and laws. However, I do not see how impartial enforcement or sound judicial interpretation of present laws can importantly affect the basic problem.

Others apparently believe that right-to-work laws might have some bearing where organization has not occurred, but they have no particular bearing where excessive employer or union power already exists.

While it is certainly desirable, the Landrum-Griffin Bill, recently passed as a result of exposure of corruption, racketeering and a host of unethical union practices, will be relatively meaningless in reducing the legalized excessive concentration of collective bargaining power.

Proposals for prohibiting union compulsion and coercion deal with important aspects of union power over the individual worker. Legislative action in this area is certainly necessary but will not substitute for that necessary to deal with the basic problem resulting from our trying to ride the competitive price horse and the monopoly wage horse at the same time. We are faced with a choice we cannot escape. The wage monopoly is destroying price competition. Competition is our means of keeping economic power divided and placing ultimate economic power in the hands of the people as consumers. Economic freedom in America depends upon the preservation of competition. The monopoly power of unions and employers for collective bargaining purposes must be divided on the basis of competition or it will destroy our economic strength and make us an easy prey for our enemies.

Then what must we do?

The answer is not the application of the antitrust laws to unions. Here are some specific proposals for amending our laws on collective bargaining that may not be the final answer but at least I think they recognize the basic problem.

1. The combining of national unions for the establishment of common bargaining demands or use of economic power should be prohibited. The same prohibition should apply to national employer organizations.

2. Unions representing more than about 10,000 employees of a single employer should be prohibited from combining to establish collective bargaining demands or exercising joint economic power against more than one enterprise. Unions should be free to combine in bargaining with employers having less than about 10,000 employees, but only within prescribed geographical limits. The same prohibition and political limitation should apply to employers and employer organizations.

3. In our basic industries, affiliated unions of a national union representing employees of a single large company, regardless of size, should be permitted to combine in their collective bargaining demands and joint use of bargaining power.

4. The law should not prohibit union cooperation aside from the above limitations.

Whatever remedy Congress may ultimately fashion to solve the problem of excessive concentration of collective bargaining power by employers and unions, I believe it must conform to these principles:

First, the full benefits of genuine collective bargaining must be retained, including the right to strike.

Second, we must avoid creating some new form of bargaining inequality.

Third, the legislation must be tailored to fit the divergent forms of trade unionism. Craft unions will require treatment different from industrial unions.

Fourth, employees working for business units having a relatively small number of employees, or for concerns operating only on a local or regional basis, must not be squeezed into a mold designed for unions representing employees of big business.

Fifth, responsibility for collective bargaining with an enterprise in a basic industry having more than about 10,000 employees should be in the hands of a union and an employer representing solely those employees.

The ultimate objective is to prevent combinations of employers or combinations of unions from destroying the competitive system by creating national emergencies. The only forms of economic discipline are competition, or some form of absolute authority. If we take the absolute authority approach of more Government control, then we are on the road to statism. To prevent the replacement of competition by absolute authority, it is necessary to prohibit joint bargaining on the part of large employers in our basic industries, as well as joint bargaining on the part of unions representing the employees of large concerns. We must also prohibit national bargaining by both unions and employers.

Now, let's turn to the concentration of industrial power. One of the factors that prevents needed action is the public misconception as to the advantages of colossal size in industry. Actually, there is some basis for union and public opinion that colossal industrial power justified colossal union power. I believe our antitrust laws, which were enacted in our early stage of industrial development, need to be modernized and strengthened to cope with new problems created by excess industrial power.

When we fail to keep our fundamental policy laws up to date, inequities arise that create pressures for special interest legislation and regulatory laws applying detailed control.

What we need is not more laws but more modern laws.

This need is pointed up by Federal subsidization to create and maintain adequate competition. During and after World War II, the Government finally broke the monopolistic position of the Aluminum Company of America by using indirect Government subsidization to establish four additional aluminum companies. In the case of the automobile industry, the President of the United States, a few years ago, directed the use of Defense Department procurement powers to indir-

ectly subsidize the continuation of one of the five remaining passenger car companies.

The competitive principle enables consumers to reward companies that successfully meet their preferences, and to reject companies that fail to do so. Elimination, or economic death, is and should be the penalty of economic failure in a competitive society. It keeps our economy healthy and responsive to consumer desire and need. However, in many of our basic industries, the competitive discipline of the market place has now reduced the number of companies to a mere handful. In the case of the automobile industry, in my opinion, the five passenger car companies left constitute barely the minimum number necessary for adequate consumer choice, confidence and discipline.

I believe preservation of the competitive principle in America is dependent upon its complete development by provision for economic birth as well as economic death in our major basic industries. I made some specific recommendations for economic birth based on the degree of market domination in such industries.

Briefly, this was what I proposed—when a company engaged in only one basic industry is doing more than 35 per cent of the business, or when a company engaged in more than one basic industry is doing more than 25 per cent of the business, such companies should be required to submit to a specified public agency their own programs for reducing their percentage of the particular business involved. An obvious way for them to do this would be through the creation of more than one company from the old company. In other words, by the process of division, or economic birth. Adherence to such percentage figures only guarantees the future existence of four or five companies in each basic industry.

This proposal would reward, not penalize, companies for being successful. It would not prevent bigness or maximum technological efficiency—it would insure a minimum number of competitors as a basis for public reliance on competition rather than government control.

When a company acquires a large share of control over a basic industry, it begins to fear the shadow of adverse government action. It must necessarily begin to restrain itself, to hobble its skill. We need all of the competitive skill we can get. We must seek to increase it, not restrain it.

My proposal for economic birth has as its objective the *further* development of competition and voluntary cooperation in this country. It is important to emphasize that the problem is not one of *size* or *bigness*. Bigness per se is not bad. Indeed bigness is essential in a modern industrial economy. Under my proposal a company would be as large as the nature of the industry required for efficient, competitive operation and no arbitrary fixed limit is suggested. The problem is the degree of economic control or power permissible in a competitive economy.

Adoption of a birth policy in industry would, I believe, again unleash some of the principal factors of industrial growth without enactment of new laws but simply the modernization of the old.

Now I would like to touch on one aspect of concentrated political power.

Big societies, especially those in competition with other big societies, certainly need strong—even big—governments and strong—even big—unions and corporations. But their power can and must be dispersed. The ultimate control can and must be in the hands of the people as citizens and as consumers. That is the only way to build the greatest strength and soundest growth.

But we cannot achieve this goal unless we recognize that it is morally, socially and economically wrong for unions as unions, or corporations as corporations, to get into politics, directly or indirectly.

It is wrong, as far as I am concerned, for the simple reason that it is wrong for any American to transfer his personal rights of citizenship to an institution—and it is wrong for an institution to act in any way to encourage the relinquishing of such rights.

Little difference in principle exists between the present excessive political influence of unions and the earlier excessive political influence of business denounced by both Roosevelts. One excess is as wrong as the other. Both are obstacles to political freedom, economic justice and individual development.

The question is whether in a democracy based on political freedom and the rights of the individual, economic organizations should be permitted to participate directly or indirectly in political affairs.

I am not referring to individual union leaders or members, or individual business leaders or white-collar workers participating in political activity. Unions or businesses certainly should not be permitted to present economic facts to political bodies or to private citizens' groups, but what I am decrying is direct or indirect political activity on the part of economic organizations, whether they are union organizations, business organizations or other organized economic groups.

Before church and state were separated, church participation in politics and government unquestionably exerted a direct influence on the political attitudes of church members. Similarly, political activity by a company is likely to cause an executive to feel that his economic opportunity in the enterprise is affected by his political support of company political thinking.

The same holds true of unions. How could union members or union officers fail to feel that their treatment by the union would be affected by conformity or opposition to union political programs?

I believe corporate officials and union officials *should* participate personally, as individual citizens, in political affairs. But I think they should take every possible step to assure members and employees that

this right is a *personal* right not to be abridged in any manner—and to encourage them to use it on that basis.

Our political parties today, unfortunately, have given victory at the polls such engulfing pre-eminence that neither party will take a position that might offend any sizable segment of voters. As a result, we have a close similarity between stated party objectives, with very little choice left to the voter. Attention is focused more on personalities, money and organization than on issues, citizen participation and spontaneous support for basic principles.

Both parties have allowed themselves, to an alarming extent, to become the captives of dominant economic groups. In Michigan, for example, one party is largely under the control of big labor, while the other is largely in the hands of big business. The dominance of these power groups, expressed through organizations rather than individual citizens, resulted in a seven-month deadlock which was broken by an increase in the sales tax, but recently declared unconstitutional—and now the stalemate is with us again, resulting in continued thwarting of progress in the state.

To combat this impossible situation, we are developing a nonpartisan citizens' program identified as Citizens For Michigan. Its members are individuals acting for themselves as citizens, not as representatives of any organization of any type.

We hope through this effort to establish a means by which an influence greater than the minority economic groups now dominating our two political parties can be created. We hope to acquaint enough citizens with the facts on the state's problems and the means for their solution to secure essential political and governmental action. Finally, we hope to restore citizen belief in the effectiveness of individual political participation.

The success of the Citizens For Michigan program can provide the basis for releasing the political parties from their captivity and restoring to the people these necessary instruments of self-government.

If we are going to encourage an increase in political participation, we must have two political parties with programs that provide the people a clear and basic choice. At least one party should be more dedicated to the principles of human liberty and justice than to political victory *at any cost* through colossal campaign expenditures, economic power, subsidy and special advantage.

To stop the drift toward increasingly massive centralized governments, we need to do three important things.

First, we must make certain, by modernized law, that power outside government is dispersed and kept dispersed.

Second, we must improve the character of state governments and modernize the smaller governmental units—the parish, the county, the

township—that were set up on the basis of travel and population distribution in a frontier country.

Third, we must revive the political participation of the *people* in control of the government.

The most effective means of putting hope and zest back into political participation would be for the political parties to reject economic power group participation, *reach out* for citizenship participation, put less emphasis on dollars and more emphasis on people and principles.

We need to bring our political parties and the people together, if our democratic system is to continue to be truly representative. When the people have little choice, they are powerless to act, and control is defaulted to wilful power groups that can be arrogantly unaccountable to a listless electorate.

As I have indicated, I don't believe the mechanical answers to excessive industry, union and government power are hard to find.

But we will not find these answers or put them into effect until we build a new fire under our aspirations as a people.

Dr. Charles Malik has said:

> You have in your own traditions certain invaluable beliefs about man and society and history and human destiny and the nature of God—beliefs that you should not be ashamed of, that you should feel free to export and to teach others. But first you must rediscover them yourselves and believe in them profoundly yourselves. Many of you have taken life too much for granted. Therefore, the great deposit of belief and conviction and interpretation of life which has come to you, you simply don't honor enough.

In considering historical fact, we have an odd propensity for believing that years must pass before the meaning of events comes into clear focus.

But listen to these lines written by Washington in 1788, during the period our new Constitution was in the process of ratification:

> The plot thickens fast. A few short weeks will determine the political fate of America for the present generation and probably produce no small influence on the happiness of society through a long succession of ages to come. Should every thing proceed with harmony and consent according to our actual wishes and expectations; I will confess to you sincerely, my dear Marquis; it will be so much beyond any thing we had a right to imagine or expect eighteen months ago, that it will demonstrate as visibly the finger of Providence, as any possible event in the course of human affairs can ever designate it.

And, a short time later:

> You will permit me to say, that a greater Drama is now acting on this Theatre than has heretofore been brought on the American stage, or any other in the World.

Can we recapture this focus? If we can, I believe it will reveal that we are much nearer than we think to a major breakthrough in human affairs.

The question in my mind is: Can we awaken and re-aim ourselves in time? Will we continue to divide excess power, or let it divide us?

We are facing two things:

Abroad, we are facing a great perversion of the ideal of human liberty, and the stain is spreading with speed.

At home, we are stalled at a crossroad—confused about our direction and split into opposing power camps. In other words, by not dividing power, we are being divided by it.

And while we are thus stalled in our continuing American revolution, the stain spreads.

To get moving without delay, we should modernize our basic anti-trust and collective bargaining laws. But we are not likely to see this change until at least one of the political parties is willing to risk putting the national interest ahead of the desire to win the next election.

The people desperately need the political leadership necessary to give them a chance to make the basic choice.

I think such an approach should and could be provided. I'm convinced it would capture the imagination and support of the American people.

While I do not anticipate that the step will be taken tomorrow, I believe it will and must be taken.

We have a certain knowledge of what has happened to others.

Defending Christianity against the charge that it had brought down the anger of the pagan gods and caused the sack of Rome, St. Augustine said: "depraved by good fortune, and not chastened by adversity, what you desire in the restoration of a peaceful and secure state is not the tranquillity of the commonwealth, but the impunity of your own vicious luxury." He reminded the Romans that even the threat of the barbarian invasion had not diverted them from their pleasures.

With this historic characteristic of peoples in mind, Somerset Maugham once wrote, "A nation that wants anything more than freedom will lose its freedom, and the irony of it is, if it is comfort and security it wants, it will lose them, too."

America's destiny is threatened but not lost. We can win by searching and risking and by sacrificing to the laws of God, our nation and our fellow man.

Helen Keller gave a magnificent summary when she said: "Not until we can refuse to take without giving can we create a society in which the chief activity of man is the common welfare."

With freedom, we can still achieve this goal. I believe we are still capable of building a new great age for ourselves and others, and thus drown out the voices of gross materialism, power and compulsion at home and abroad that would destroy us.

MINUTES OF THE ANNUAL MEETING

January 20, 1960

The Annual Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 150 members and guests in attendance.

The President stated that the minutes of the Stated Meeting of November 18, 1959 were published in the December issue of the JOURNAL. There being no additions or corrections the minutes were approved as published. He then announced the minutes of the Stated Meeting of December 16, 1959 will be published in the next issue of the JOURNAL and will be presented at a later Stated Meeting of The Institute for approval.

Mr. LePage then called on Dr. Joseph S. Hepburn, Chairman of the Tellers of Election, for the report of the tellers who had been appointed in accordance with the By-Laws to count the votes for ten Managers to be elected. Dr. Hepburn reported that 2867 votes were cast of which 2469 were legal and 398 defective, and the nominees received votes as follows:

Brandon Barringer	2207	Richard T. Nalle	2223
Edward G. Budd, Jr.	2288	S. Wyman Rolph	2213
J. Frank Cox	2247	John Russell, Jr.	2219
Joseph Gray Jackson	2230	W. Maxwell Scott, Jr.	2219
Ezra S. Krendel	1507	I. Melville Stein	1908
James McGowan, Jr.	2173		

The President then declared the following ten members were thereby elected Managers of The Franklin Institute for a term of three years:

Brandon Barringer	Richard T. Nalle
Edward G. Budd, Jr.	S. Wyman Rolph
J. Frank Cox	John Russell, Jr.
Joseph Gray Jackson	W. Maxwell Scott, Jr.
James McGowan, Jr.	I. Melville Stein

Mr. LePage thanked Dr. Hepburn, Mr. Kane and Mr. Neher for their services as Tellers of Election.

The President reported that at the Board of Managers' meeting this afternoon, Mr. J. G. Richard Heckscher had been elected as Executive Vice President of The Franklin Institute, effective February 1, 1960. Mr. LePage said that Mr. Heckscher had been a member of The Franklin Institute since 1934 and a member of its Board since 1952, and he was delighted to make this announcement at this Annual Meeting of The Institute.

The President called upon the Secretary to read the report submitted by the Trustees of the Elliott Cresson Medal Fund showing the financial transactions for the year, which, in accordance with the Deed of Gift that forms the basis of the Award, must be presented at the Annual Meeting of The Franklin Institute. The report follows:

ELLIOTT CRESSON MEDAL FUND

Report of the Trustees to The Franklin Institute

January 20, 1960

PRINCIPAL

Principal of the Fund (Representing the original $1,000 plus transfers thereto from Unexpended Income and profit from disposition of investments)		$4,304.73
Invested as follows:		
$4,000 U. S. Treasury 2½% bonds, due 6/15/1962–67 at book value	$4,039.57	
Cash in Western Savings Fund Society	265.16	
	$4,304.73	

INCOME

Income of the Fund for 1959		
Interest on bonds (less amortization)	$ 126.29	
Interest on bank balance	7.76	$ 134.05
Expenses		
Medals, certificates, insurance and cases		329.55
Expenses in excess of Income for the year 1959		(195.50)
Unexpended Income at January 1, 1959		2,166.99
Unexpended Income at December 31, 1959		$1,971.49
Invested as follows:		
$2,000 U. S. Treasury 2½% bonds, due 6/15/1962–67 at book value	2,019.89	
Cash in Fidelity-Philadelphia Trust Co.	24.65	
	$2,044.54	
Unpaid expenses because of unavailable cash	73.05	
	$1,971.49	

/s/ *Henry B. Allen*
HENRY B. ALLEN

/s/ *Delmer S. Fahrney*
DELMER S. FAHRNEY
Trustees

The President presented a summary of the Annual Report of The Franklin Institute for 1959 and stated that this Annual Report would be published in the April issue of the JOURNAL.

The President then announced he took considerable pleasure in introducing the speaker of the evening, Dr. Chauncey D. Leake, President, American Association for the Advancement of Science and Dean, College of Medicine, The Ohio State University. In speaking of the title of his talk, "Our Unbalanced Biad," Dr. Leake said that the word "Biad" would not be found in any dictionary—that the word expressed that there are two arms of our intellectual balance—the humanities on one hand and the sciences on the other, and it is our continuing job of learning to keep these long arms of intellectual effort in balance. Dr. Leake said that today we are witnessing a new resurgence of enthusiasm for understanding science, for achieving worthy purposes in human welfare, and in preserving the natural beauties of our world, even in our increasing industrialization. After a full discussion period, Mr. LePage expressed very grateful appreciation for Dr. Leake's visit to The Institute and for presenting a brilliant lecture.

The meeting was adjourned at 9:40 P.M. with a rising and enthusiastic vote of thanks to Dr. Leake by the audience.

WILLIAM F. JACKSON, JR.
Secretary

MEMBERSHIP

The following elections were approved on January 7, 1960 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Malcolm J. Baber
Joel N. Bloom
William A. Bourse, III
Edward P. Buckley
Charles A. Cherricole
John E. Christ
Thomas G. Conley
Lloyd R. Coon
Dante DiGaetano
Frank J. Donahoe
Howard R. Doty
E. Jay Ferry
David Gelfand
Leonard Goland
Joan S. Guldemond
Paul H. Griffith
Allan C. Hanson
Donald B. Houghton
John A. Howsman
Russel F. Hurst
Frank T. Innes
John J. Jackson
Mortimer M. Labes
Joseph R. Ladd
Abraham L. Levine
Alfred M. Loeb
Donald G. Maier
Jack H. Martinez
William K. McLeod
Mrs. Robert R. Meigs
E. Everet Minett
Antoinette R. Montanaro
Aloysius Joseph Planeczky
Paul R. Pollock
Walter E. Powell, Jr.
Joseph James Prochazka
Manuel Raefsky
Charles Randall
John F. Reardon
Mark Reber, Jr.
Charles W. Reed
Claude E. Reeder
Homer R. Reese
Nicholas Reitter, Jr.
Walter J. Ricketts
Francis W. Ringer
Fred J. Roberts
Alfred F. Roman
Alvin Rosen
Walter E. Rosengarten, Jr.
Raymond W. Ross
Robert G. Ruch
Merritt A. Rudner
August F. Sachs
Pedro G. Salom, Jr.
William C. Sampson
Roy J. Scheetz
Robert J. Schlosser
H. Orvel Sebring
John J. Seelaus
Robert Sehr
Edward Lowber Stokes
Paul M. Tunison
Ralph Weinger
John W. Wyld

EDUCATOR

Mrs. Marvel Chang Chun
Mrs. Edgar A. Erickson
Theodore M. Hesser, Jr.
Schuyler V. Jenkins
Joseph F. O'Connor
J. Robert Snyder

NON-RESIDENT

Robert Kyracou

NECROLOGY

Oliver E. Buckley '26
Sterling H. Bunnell '13
Edward S. Dillon '42
W. G. Ellis '24
C. D. Galloway '22
J. Aubrey Hope '53
Abigail C. King '36
Harry Langsam '45
Fritz S. Poll '52
Mrs. John B. Townsend '34
B. G. Waring '36
C. T. R. Wilson '29

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, January 13, 1960.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 13, 1960.

DR. WILLIAM G. SCHMIDT in the Chair.

The following reports were presented for final action:

No. 3332: Work of Hugh L. Dryden in Aerodynamics and Guided Missiles.

This report recommended the award of an Elliott Cresson Medal to Hugh Latimer Dryden, of Washington, D. C., "In consideration of his many scientific and practical contributions to the theory and application of aerodynamics which greatly advanced the art of wind tunnel and aircraft design and for his guidance of and personal contributions to the design and development of the world's first automatic radar homing guided missile."

No. 3365: The Boyden Premium.

This report recommended the award of The Boyden Premium in the amount of five hundred dollars to Carl I. Aslakson, of Bethesda, Maryland, "In consideration of his contribution to the measurement of the speed of radiation in space through the use of Shoran techniques and thereby as the first American to aid in establishing a new and significantly more nearly accurate value of 16 km. per second higher than the long accepted value."

D. S. FAHRNEY,
Secretary to Committee

YOU CAN ADVANCE SCIENCE EDUCATION

Today, more than ever before in its 135-year history, there is vital need for The Franklin Institute to effectively promote education in science and technology. It is imperative that we meet this challenge by providing adequate educational opportunities in these fields. This requires vision, objective planning, and money. We have more than enough of the first two requisites, but far too little of the third.

Our programs are aimed at professional scientists and industry, as well as the lay public and young people seeking inspiration and guidance in choosing a career. The Institute's educational programs are impressive, for they begin with students in the early grades of our elementary schools and continue throughout an individual's professional or industrial life. With more funds at our disposal, the scope and vigor of these activities could be greatly increased.

The Franklin Institute is not richly endowed. It is a non-profit organization, depending for encouragement and support on an understanding public. Capable and conservative management assures wise administration of all funds.

Your gift or bequest, large or small, will be deeply appreciated and will be used effectively to broaden the Institute's educational usefulness. There is a warm satisfaction in giving financial support to an organization that has pioneered in, and is dedicated to, the advancement of science and technology through education.

When property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

The Secretary of The Franklin Institute will gladly furnish you with additional information. Write to him at The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

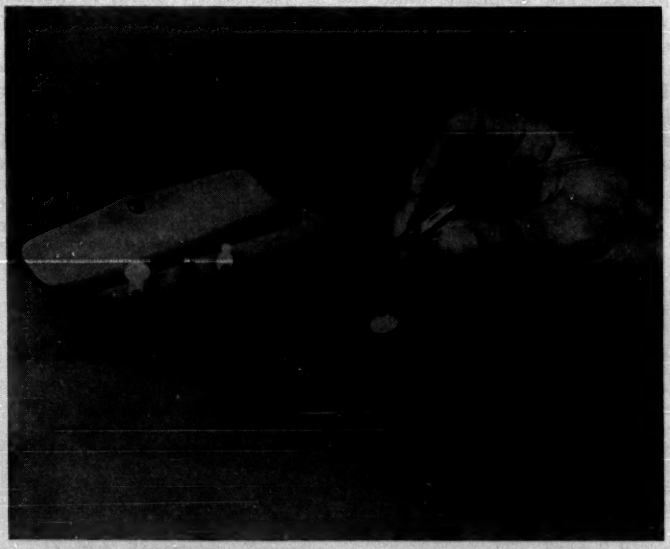
"LAMPREY"

BY

C. W. HARGENS[1]

A useful instrument for numerous laboratory and industrial occupations is the "Lamprey." Nicknamed for the "eel-like parasitical fish with a circular suctorial mouth," this development was conceived as a new product to be introduced by a sponsor who has found similar devices useful in biological work.

The instrument uses a vacuum principle with an arrangement of carefully designed chambers, orifices, and nozzles to pick up delicate objects with precise control. Held as illustrated in the figure, the instrument is controlled by the index finger. When the control port is closed by the finger, a strong lifting force results at the tip of the instrument; removal of the finger causes immediate release of the object being lifted.



[1] Head of The Franklin Institute Laboratories' Bioelectronics Branch.

Very thin mica leaves used in electric capacitors, microscope cover glasses, and similarly flat objects which cannot be handled successfully with tweezers work well with the vacuum device. There are in addition those biological specimens which should not be handled by warm, damp, rough and inaccurate fingers. The "Lamprey" also eliminates chemical damage to surface-sensitive materials which come in contact with the oil and acid normally present on finger tips. Man's fingers, tipped with claws, have been generally practical for daily living, but they are crude instruments in many modern technological situations.

Some objects, because of contamination, radioactive, chemical, pathogenic or otherwise, should not be handled. The instrument contains a filter which can take various forms to protect the vacuum system from the input of foreign matter. The nozzles shown are a few of the readily interchangeable types. These particular ones are of stainless steel with tip cones of Teflon, a unique plastic which does not adhere to "gummy" surfaces. An important advantage of the "Lamprey" is that it can be easily disassembled for thorough cleaning.

The instrument was designed for production by several methods. Precision models for exacting tasks can be made of high-quality metals, or versions using this very same design are capable of being molded in plastic. The all-plastic units can be particularly useful in working with corrosive products. In some applications they may be considered expendable, to be discarded after a certain amount of use.

This small project is an example of product development in the Institute Laboratories. The client in this case was not a large concern but an individual with a proprietary business interest. The ideas presented were quickly and economically translated into a practical design and working prototypes.

BOOK REVIEWS

THERMODYNAMIC AND TRANSPORT PROPERTIES OF GASES, LIQUIDS AND SOLIDS, sponsored by ASME. 472 pages, diagrams, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $12.50.

This volume is a collection of forty-two papers presented during a symposium on thermal properties held at Purdue University, Feb. 23–26, 1959. The symposium was sponsored by the Standing Committee on Thermophysical Properties, Heat Transfer Division of the American Society of Mechanical Engineers. Increasingly of late, conference proceedings are being published between hard covers. This is probably a boon to libraries, which are faced with problems of handling and storage, but the practice makes it an economic impossibility for the private professional man, who might otherwise obtain a copy for the sake of a few papers, to purchase his own copy. And, here, as in many conferences, the papers treat so many diverse topics within the broad general area indicated by the title that it seems hardly likely any one man would find even half the papers particularly useful. For example, the papers range from the thermal conductivity of milk and the viscosity of steam to the combustion of boron hydrides and the unmixing of metal solutions by a temperature gradient. The papers are grouped into nine sessions.

I. Theoretical Estimation of Transport Properties
II. Review of Recent Work on Transport Properties
III. Thermodynamic Properties—Gases and Liquids
IV. PVT Data and Equations of State
V. Thermodynamic Properties of Boron Compounds
VI & VII. Transport Properties: Experimental
VIII. High Temperature Transport Properties of Metals and Ceramics
IX. High Temperature Thermodynamic Properties of Gases

Recommended for technical libraries, but not a good buy for individuals.

F. J. DONAHOE
The Franklin Institute Laboratories

STANDARD HANDBOOK FOR TELESCOPE MAKING, by N. E. Howard. 326 pages, illustrations, 6 × 9 in. New York, Thomas Y. Crowell Co., 1959. Price, $5.95.

Astronomy has taken to the front page in the daily papers and the desire for telescopes has reached a fairly feverish pitch. Amateur telescope making continues to spread as rapidly as the know-how can be distributed. Most of the present literature has been criticized for various reasons. Perhaps this book may be the answer to your problem.

Standard Handbook for Telescope Making lives up to its title. A chapter on the history of the optic tube and why amateurs make reflectors sets the stage for the how-to-do remainder of the book.

This exposition, written by a school teacher, is explicit as to what is desired and how to obtain the proper results. Setting parallel methods for obtaining the required finish. Step by step direction for mirror, diagonal, eyepiece, mounting, clock-drive, observatory, photography plus enough astronomy to get one into the observing habit. The illustrations are almost sufficient in themselves.

A little friendly criticism might be in order. The chapter on eyepieces would be much better with instructions for making small lenses included. The bibliography at the end of this chapter does give the books containing small lense practice but this still leaves this as the only weak chapter. Using carborundum for grinding optics, cutting lap grooves with a hand saw and painting the pitch lap with bees wax are features that are better left out of an amateur instruction book. With these few details corrected, this book would really serve as a one book library for this interesting scientific sport.

EDWIN F. BAILEY
The Fels Planetarium

PARIS SYMPOSIUM ON RADIO ASTRONOMY, edited by Ronald N. Bracewell. 612 pages, illustrations, 6 × 9¼ in. Stanford, Stanford University Press, 1959. Price, $15.00.

In 1932 Dr. Karl G. Jansky, of the Bell Telephone Company, recorded some strange radio noises in the sky. While this was a new phenomenon, scientists were not overly excited about it and more than 9 years elapsed before Grote Reber built his old fashioned radio telescope in Wheaton, Illinois, to pursue these radio sources in the sky.

The art was neglected during the war with other matters occupying the attention of astronomers and scientists. However, as a result of technological developments during the war a tremendous wealth of material was available for use by scientists. When this equipment was turned on the sky, a new science was born—radio astronomy.

While it began very slowly with astronomers feeling their way and gingerly contemplating installations—they are frightfully expensive—the pace in this field accelerated until today it is looked upon as a new powerful tool of the astronomer. Light recorded by instruments is scattered, absorbed and attenuated by space and by the atmosphere of the earth. Thus, when the astronomer points his telescope to certain parts of the sky an impenetrable curtain exists. However, this powerful new tool is able to penetrate to unheard of distances and, as a result, man's concept of the universe—its size, shape and structure—may be amenable to solution.

Today there appears a comprehensive book unfolding the story of the advances in this new science. The need for a gathering of astronomers and specialists in this field was recognized in 1955 at the IAU (International Astronomical Union) meeting in Dublin. Three years later this meeting was held in Paris under the joint sponsorship of the IAU and ISRU (International Scientific Radio Union.) In attendance were the ranking men in astronomy. These men gave papers on various facets of this new science.

This book represents the symposium proceedings. The foresight of the planners made certain that full coverage would be achieved. Under the chairmanship of J. L. Pawsey of the Radiophysics Laboratory of Sydney, Australia, a well balanced program was laid out and speakers were invited to present papers to the symposium.

The subject was divided into six sections:

I Radio emission and reflections from planets, comets and the moon.
II Solar radio emission and the quiet and active sun.
III Radio study of individual objects (external to solar system).
IV Radio evidence on the large-scale structure of our own and external galaxies.
V Source survey, identifications, and other studies related to cosmological problems.
VI Theory, mechanism of solar and cosmic radio emission.

In the 107 papers presented at the symposium are to be found every facet of this new science under investigation today. This new book represents the current state of the science as of 1959.

I. M. LEVITT
The Fels Planetarium

BOOK NOTES

THE CORRESPONDENCE OF ISAAC NEWTON. VOLUME I, 1661–1675, edited by H. W. Turnbull. 468 pages, plates, illustrations, 8 × 11 in. New York, Cambridge University Press, 1959. Price, $25.00.

The present work represents the culmination of over fifty years of hoping and planning for a standard critical edition of Newton's collected writings. The first few attempts were not productive; but since the Newton Letters Committee of the Royal Society was reconstructed under the chairmanship of Prof. E. N. da C. Andrade in 1947, the work has been progressing steadily. Volume I appeared late in 1959 and it is hoped that by 1960—the 300th anniversary of the Royal Society—other volumes will be available.

This first volume contains 156 manuscripts and letters written during the period 1661 through February 1676, covering not only Newton's own writings, but others relative to his correspondence. The excellent "Introduction" by Prof. Andrade reveals the scope of the undertaking and the difficulties and problems relevant to the publication of the work. In the "Preface," Prof. Turnbull calls attention to the contents, the arrangement and selection of material, and several previously unpublished items.

The book is beautifully printed and bound. The index to Vol. I is well arranged and quite complete. The high standards set by Vol. I indicate that the complete work will be a major contribution to modern scholarship and the history of science. When completed, the long-needed, definitive series will be a most welcome addition to the reference libraries of the world.

SCIENCE & STATE GOVERNMENT, by Frederic N. Cleaveland. 161 pages, diagrams, 5½ × 8½ in. Chapel Hill, University of North Carolina Press, 1959. Price, $3.50.

In 1954, the National Science Foundation authorized a study of the role of state governments in scientific activities, to be carried out by the Institute for Research in Social Studies at the University of North Carolina. This small volume reports on six states representing diversity in population, economic characteristics, size, geographic location and political environment. Expenditures of public funds are analyzed in four major fields—agriculture, resources and public works, health and welfare, and higher education. Also described are the administrative aspects of these programs, intergovernmental relations, and the scientists' place in the state. A final chapter on "Summary and Conclusions" sums up the findings and points out the similarities. It is interesting to note that in four of the six states, agriculture claimed the highest portion of state funds and was second highest in the other two states.

TRANSFORMERS FOR THE ELECTRIC POWER INDUSTRY, by Richard L. Bean, Nicholas Chackan, Jr., Harold R. Moore and Edward C. Wentz. 409 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $12.50.

The authors, all of the Transformer Engineering Department at Westinghouse, have prepared a practical manual on transformer design, construction, application, operation and maintenance. In addition to the standard material (equations, tables, empirical data), the book contains new material on the economics of transformers, the latest advances in methods of temperature calculation, and new testing techniques. Chapters 3 through 8 emphasize theory; Chapters 9 through 14 are of more interest to engineers who operate the transformers.

ELEMENTS OF MATERIALS SCIENCE, by Lawrence H. Van Vlack. 528 pages, illustrations, 6 × 9 in. Reading (Mass.), Addison Wesley Publishing Co., Inc., 1959. Price, $8.75.

This is an elementary text for sophomore and junior students of engineering, covering structure of metallic and nonmetallic materials. It aims to give a background for subsequent engineering design and analysis courses. A knowledge of introductory college mathematics is assumed.

ANALYSIS OF ELECTRIC CIRCUITS, by Egon Brenner and Mansour Javid. 703 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.50.

This introduction to linear electric circuit analysis on the sophomore-junior level utilizes the reader's background in calculus, but does not introduce advanced mathematics. Many problems and numerical examples are included in this text.

ORGANIC CHEMISTRY, by Donald J. Cram and George S. Hammond. 712 pages, diagrams, illustrations 6½ × 9¼ in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $8.50.

This introductory text in the field of organic chemistry is intended primarily for chemistry majors, premedical students, engineers, etc., who require a working competence in the subject.

MODERN NETWORK ANALYSIS, by Fazlollah M. Reza and Samuel Seely. 373 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $10.00.

This text, on the intermediate level, provides the reader with an understanding of the general properties of networks, both in the steady state and the transient state. Each chapter is provided with numerous carefully prepared problems for exercises.

THEORY AND DESIGN OF SMALL INDUCTION MOTORS, by Cyril G. Veinott. 477 pages, diagrams, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959 Price, $13.50.

In this practical guidebook, the author thoroughly covers the application, engineering and design procedures necessary when working with small induction motors. Divided into four sections for easy reference, this text treats theory and performance calculation methods, provides methods and formulas for analyzing a given design and gives practical design procedures. Emphasis is mainly upon the basic theory of induction motors and mathematics is used only as needed to explain methods and develop formulas.

AUTOMATING THE MANUFACTURING PROCESS, by George F. Hawley. 147 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $4.95.

In this text, the author presents in practical terms some of the mechanical problems involved in inventing and designing machinery for specific operations, such as assembly of components, packaging of individual products, bottle capping, feeding, etc. This volume may help management and production personnel gain a greater appreciation of the risks encountered in successful automation, while designers and development engineers will find ideas which should minimize their failures.

ENGINEERING MECHANICS: STATICS, by Irving H. Shames. 303 pages, diagrams, 6 × 9 in. Englewood Cliffs, Prentice-Hall, Inc., 1959. Price, $6.35 (trade); $4.75 (classroom).

As engineering gradually upgrades itself to what not so many years ago would have been called theoretical physics, the education of engineers advances accordingly. The present book is one of a number of recent efforts to elevate the old subject of statics by generalizing its concepts, adding topics usually encountered later, and treating everything in terms of vector algebra. In this case, the author adds chapters on the elements of continuum mechanics, including the concept of the stress tensor, and an introduction to variational methods through virtual work. This new material is treated in a thorough and intelligible manner, and comprises more than a quarter of the book. Illustrations and makeup are excellent, and many problems have a topical flavor for added student interest.

ADVANCES IN ELECTRONICS AND ELECTRON PHYSICS, VOL. XI, edited by L. Marton. 523 pages, illustrations, 6 × 9 in. New York, Academic Press, Inc., 1959. Price, $15.00.

This eleventh annual volume follows the pattern of the previous volumes, reviewing recent advances in various phases of electronics and electron physics. The topics covered in Vol. XI are: photoemission (Görlich); parity nonconservation (Stern-

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heimer); efficiency of detectors for infrared radiation (R. Clark Jones); automatic data processing (Barlow, Ovenstone and Thonemann); operational amplifiers (Knoigsberg); telemetering (Riblet); electron-diffraction method for semiconductors (Pinsker); and secondary emission from solids (Hachenberg and Brauer). The list of contributors reflects the friendlier climate between Eastern and Western scientists.

AMINO RESINS, by John F. Blais. 220 pages, illustrations, 5¼ × 7¾ in. New York, Reinhold Publishing Corp., 1959. Price, $4.95.

This little volume covers the chemistry, manufacture and major applications of urea and melamine resins, and the newer ethylene urea and benzoguanamine. Described in detail are amino resin applications in adhesives, molding laminating, textiles, paper and surface coatings. Previously only a limited amount of information was available on this resin group; this book provides additional data.

ANNUAL REVIEW OF NUCLEAR SCIENCE, VOL. 9, edited by E. Segrè and L. I. Schiff. 625 pages, diagrams, 6 × 9 in. Palo Alto, Annual Reviews, Inc., 1959. Price, $7.00.

Volume 9 in this useful series contains fifteen papers contributed by nineteen authors. The subjects cover a wide range of nuclear science, including photodisintegration, technetium and astatine chemistry (with emphasis on the period since 1948), high temperature plasma research, vertebrate radiobiology and cellular radiobiology. L. Okun', from the Academy of Sciences in Moscow, has an interesting paper on "Strange Particles: Decays," in which he discusses leptonic decay and weak nonleptonic interactions.

THE SUN, by Karl Kiepenheuer. 160 pages, illustrations, 5 × 8¼ in. Ann Arbor, University of Michigan Press, 1959. Price, $5.00.

Aimed primarily at the layman, this compact book presents a clearly defined picture of the astronomer's sun. Translated from the German by A. J. Pomerans, the ten chapters examine the many facets of the turbulent solar atmosphere. How man's environment is controlled almost entirely by the sun, its layered atmosphere, solar phenomena, the sun's rotation, radio waves, eclipses, and solar spectrographs are among the topics thoroughly discussed in the author's clear and concise style. This up-to-date text provides important data about the behavior of the stellar universe at large.

PUBLICATIONS RECEIVED

READING GERMAN FOR SCIENTISTS, by Hans Eichner and Hans Hein. 207 pages, 5½ × 8¼ in. New York, John Wiley & Sons, Inc., 1959. Price, $5.25.

ELECTRICAL ENGINEERING FOR PROFESSIONAL ENGINEERS' EXAMINATIONS, by John D. Constance. 448 pages, diagrams, 5¼ × 8 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.50.

HANDBOOK OF AUTOMATION, COMPUTATION AND CONTROL, edited by Eugene M. Grabbe, Simon Ramo and Dean E. Wooldridge. Volume 2, Computers and Data Processing. v.p., diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $17.50.

PETROLEUM GEOLOGY, by Kenneth K. Landes. Second edition, 443 pages, illustrations, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $9.50.

THE CATHODE RAY TUBE AND ITS APPLICATIONS, by G. Parr and O. H. Davie. Third revised edition, 433 pages, illustrations, 5½ × 8½ in. New York, Reinhold Publishing Corp., 1959. Price, $9.50.

FOUNDATIONS OF AERODYNAMICS, by A. M. Kuethe and J. D. Schetzer. Second edition, 446 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $11.75.

HANDBOOK OF ELECTRONIC CONTROL CIRCUITS, by John Markus. 347 pages, diagrams, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $8.50.

GERMAN SECRET WEAPONS OF WORLD WAR 2, by Rudolf Lusar. 264 pages, illustrations, 5¼ × 8½ in. New York, Philosophical Library, 1959. Price, $10.00.

THE WORLD'S LIGHTHOUSES BEFORE 1820, by D. Alan Stevenson. 310 pages, illustrations, 8½ × 11 in. London, Oxford University Press, 1959. Price, 63 shillings (U.K. only).

SIXTH INTERNATIONAL CONGRESS AND EXHIBITION OF ELECTRONICS AND ATOMIC ENERGY, U.S. PAPERS. 81 pages, illustrations, 8 × 10½ in. Oak Ridge, U.S. Atomic Energy Commission, 1959, Price, $2.25 (available from Office of Technical Services, Dept. of Commerce, Washington 25, D. C.).

THE REAL NUMBER SYSTEM, by Grace E. Bates and Fred L. Kiokemeister. (A reference designed to be used with "Calculus with Analytic Geometry," by the same authors, second edition.) 82 pages, diagrams, 6 × 9 in. Boston, Allyn and Bacon, Inc., 1960. Price, $1.50 (paper) (tentative).

Optics and Spectroscopy. The Optical Society of America, with a grant-in-aid from The National Science Foundation, has begun translation of the Russian journal of this name with the January 1959 issue. Subscription rates for various classifications of members and non-members of the Optical Society are available from the Society's Executive Offices, 1155 Sixteenth St., N.W., Washington 6, D. C.

ORGANIC SYNTHESES, VOL. 39, edited by Max Tishler. 114 pages, diagrams, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $4.00.

CURRENT TOPICS

Rocket's Exhaust Heat Turned to Electric Power.—A radically new electronic generator with no moving parts, designed to convert the heat of a rocket exhaust directly to electric power to run steering controls of a rocket or guided missile, has been announced by the Radio Corporation of America and the Hunter-Bristol Division of Thiokol Chemical Corporation.

The new device, a developmental RCA tube of the type known as a thermionic generator, has already produced up to 270 watts of power directly from a high-temperature heat source—an output of nearly 80 watts per pound of its 3½-lb. total weight. Coupled to a Thiokol solid-fuel rocket motor in a special test mount, the tube has generated useful levels of power directly from the rocket exhaust heat during preliminary evaluation tests.

According to RCA and Thiokol engineers, the results of the tests with both simulated and actual rocket heat sources have shown the feasibility of using thermionic tubes as efficient and light-weight generators to produce the electric power required in large missiles and satellite launching vehicles during the critical launching and upward flight stages. They pointed out that such devices could power the steering mechanism of the rocket as well as the guidance, telemetering, and related electronic equipment of a missile throughout the operating life of the rocket motor.

Commenting on the advantages of the thermionic generator tube in the rocket application, Dr. Karl G. Hernqvist who headed the group which developed the tube, said:

"A light-weight power source of this type, operating directly from exhaust heat that is normally wasted, represents a long forward step in one critical area of missile and space technology."

The present experimental RCA tube has the form of a hollow cylinder. In the Thiokol test facility, it slips like a sleeve over the rocket-flame tube through which the burning fuel passes. The method of operation was described this way by Dr. Hernqvist:

The device bears a family resemblance to conventional two-element electron tubes, or diodes, in having a cathode from which electrons are "boiled" by the application of heat, and another electrode to which these electrons flow to produce an electrical output.

In the experimental tube for rocket application, the cathode is formed by the inner wall of the hollow cylinder, while the second electrode is formed by the outer wall. Between the two walls is a narrow space filled with cesium vapor.

When the cathode wall is heated by the burning rocket fuel in the flame tube, electrons are "boiled" out of the cathode material and into the space between the two walls. The cesium vapor that occupies this space becomes ionized upon contact with the hot cathode, and encourages the easy flow of electrons to the outer wall. The electrical power that is generated by this process is fed to the rocket steering control mechanism or to electronic apparatus by means of cables attached to the cathode and the second electrode.

Dr. Hernqvist pointed out that the size and shape of the experimental tube were designed specifically for the Thiokol test installation in order to study the feasibility of the system. He said that the design of eventual operating types of thermionic generator tubes would vary in accordance with the various designs of the rockets in which they might be used. He added that the operating principles of the tube are identical to those demonstrated in earlier experimental thermionic generator tubes developed at RCA Laboratories for the direct conversion of the sun's heat to electrical energy.

Instrument Magnifies Light 50,000 Times.—It is now possible to give details of the "Image Intensifier" recently developed in Britain and described as "an exceptional breakthrough" in the field of optical imagery. Created at the Imperial College of Science in London, the device intensifies light 50,000 times and illuminates optical images invisible to the naked eye to the point where they can be seen and photographed. While the Russians are believed to have something similar, this is the first such device developed in the Western world. American scientists from Mt. Wilson and Palomar will soon be given the opportunity to see the intensifier in action. It is expected to have particular application in astronomy, nuclear physics and hospital X-ray examinations.

The ability to photograph sky bodies by almost instantaneous "snapshot" exposures through the intensifier would largely overcome the problem of local disturbances in the earth's atmosphere encountered with lengthy telescope-exposure and open up the prospect of examining the spectra of some of the faintest stars.

In nuclear physics the instrument is to be tried in conjunction with a new method of observing the geometrical shape of the track of nuclear particles. In hospitals, after further development, it may greatly reduce the exposure needed in X-ray examinations, thereby lessening the hazards of radiation.

The instrument itself is original not in concept but in method of construction. Designed to multiply the brightness of a faint image, it effectively combines multiplication of light with adequate definition. It can turn the smallest pulse of light energy into a flash that is visible and the image into a pattern that is discernible.

The intensifier comprises a glass vacuum tube about 18 in. long by 2 in. in diameter. It operates by focusing the original faint picture onto a light-sensitive cathode inside it. The cathode emits electrons which are passed, successively, through five dynodes situated along the length of the tube. Conducting coils surrounding the tube produce a magnetic field that acts as a guide to the electrons within. The cathode is made of a thin layer of an antimony and cesium compound (obtained by evaporating the cesium in a vacuum onto a thin film of the antimony), which acts as a semi-conductor. From every twenty or so photons (or atoms of light) which fall on this material, one electron is produced. Each electron from the photocathode hits each successive dynode at speed, being accelerated along its course by more than a thousand volts. At every dynode, each will produce some five new electrons, and these in turn hit the next dynode—to be again multiplied. At the end, at least 3000 electrons are produced for each one originally leaving the cathode. A final accelerating electric field throws them against a fluorescent screen, and a visible flash of light is produced.

The geometrical optics of the tube are good enough for details in the pattern on the cathode about one fifteenth of a millimeter in diameter to be reproduced clearly on the screen.

The present area of the dynodes is small—just smaller than a dime. It is planned to enlarge them in future instruments.

Even with these small pictures, detail is on a par with a 250-line scanning TV system. With bigger dynodes, performance approximating a 600–700 line scanner could probably be achieved.

"Varactor" Diode Used in Radio Receiver.—An ultra-high frequency mobile telephone radio receiver designed around solid-state devices was described by L. G. Schimpf of Bell Telephone Laboratories at the annual meeting of the Professional Group on Vehicular Communications of the Institute of Radio Engineers. Mr. Schimpf told how new devices such as the "varactor" diode can be used in a solid-state radio receiver designed to operate at around 900 mc.

The receiver is of the double-conversion superheterodyne type with a first i.f. centered at 63 mc. and the second at 10.7 mc. It has a sensitivity of about one microvolt for 13 db of quieting. All amplification, conversion, and frequency generation operations are fulfilled by using solid-state devices.

To provide the high local oscillator frequency necessary for the first conversion, a three-step chain is employed, consisting of a crystal-controlled transistor oscillator, a transistor doubler and a frequency tripler. It is in this frequency tripler stage that the special characteristics of the varactor diode are employed to advantage.

The "varactor" diode, or variable-capacitance diode, was developed recently at Bell Laboratories. It consists of a p-n junction whose capacitance varies as the reverse voltage across the junction is varied. Thus, if a sine wave is impressed on the diode, the output will be very rich in harmonics. The varactor diode is, therefore, a very efficient harmonic generator and in the multiplier chain functions as a frequency tripler.

A silicone diode is used as the first mixer, and the first i.f. consists of a two-stage transistor amplifier with a gain of about 35 db. The second oscillator is transistorized and crystal-controlled, providing a frequency of 73.7 mc. This frequency, when mixed with the first i.f. of 63 mc., gives the second i.f. of 10.7 mc. This second i.f., which is also transistorized, serves as a limiter when FM signals are being received.

The varactor diode used in this receiver is a versatile device. It has been employed extensively as the variable reactance element in parametric amplifiers to provide extremely low-noise radio-frequency amplification.

TV Optical System for Dental, Medical Use.—A new instrument that permits a dentist to view any part of a patient's mouth, highly magnified, on a TV screen has been successfully demonstrated at the U.S. Navy Dental School, Bethesda, Md., according to a joint announcement by the Navy and Avco Corporation.

The device, which is a new optical fiber probe linked to a closed-circuit TV system, can eventually be adapted for medical probes to explore inside the human body. It has been developed under a feasibility study sponsored by the Office of Naval Research and carried out at Wilmington,

Mass., by Avco's Research and Advanced Development Division.

TV cameras are now used in very limited dental application, since they can view only part of the mouth, they present a difficult lighting problem and cause discomfort to both patient and dentist because of their size.

The optical probe system being developed at Avco consists of a bundle of optical fibers bound together in a small whip-like cable with a fingertip-size lens arrangement at the probing end and coupled to a closed-circuit TV camera at the other end. A bundle contains up to 10,000 of the hair-like fibers. Each tiny fiber picks up light from a microscopic section of the surface in front of it and transmits the speck of light to the other end. With thousands of the fibers bound tightly into a cable, a picture made up of the thousands of light segments can be sent from the lens inside the patient's mouth to the TV camera and from there to the screen, where the tooth, or a portion of it, is seen magnified up to 35 times its actual size.

This permits the dentist to display selected portions of the mouth or individual teeth during actual operative procedures, for simultaneous inspection by other dentists or by students. From a training standpoint, it would permit large classes of students to watch the procedures of an experienced practitioner.

Successful use of the new system in "endoscopes"—the medical term for probes used to explore inside body cavities—would be a great step forward over any current technique. The flexible probe could reach and illuminate areas not now accessible. It could display these areas, highly magnified and in color, for inspection by several specialists for immediate consultation and diagnosis. It could render surgical operations highly visible to students, who now must rely almost entirely on the surgeon's running commentary for an understanding of the operation.

Since the fibers can transmit light in both directions, they could also be used to bring in light from a remote source without electrical wires. This means that body cavities could be illuminated for inspection or treatment without the hazard of electrical sparks in the presence of an explosive anaesthetic.

The use of fiber optics for transmitting light has been under exploration at various places in the United States and Europe for many years, and their application to medical probes has also been previously investigated. The Navy-supported Avco feasibility study, however, is the first to couple the fibers, lenses and a TV camera, to obtain a system permitting great magnification and simultaneous viewing.

High-Voltage Electricity from Sunbeams.—A new device that can produce high-voltage electricity directly from the warming power of a sunbeam or other heat source has been announced by International Telephone and Telegraph Corporation. Relying on the inexhaustible supply of solar heat, the simple system for converting the sun's rays to electrical energy is expected to have far-reaching effects in space communications and space travel. Called a ferroelectric converter, the device was announced at the 14th Annual Meeting and Astronautical Exposition of the American Rocket Society.

Unlike solar batteries which supply only direct current at low voltages, the new development supplies both a-c. and d-c. current at high voltages. Solar batteries depend on light for their usefulness. The ITT converter may use other heat sources, such as nuclear

heat, and, therefore, can operate while in shadows.

In sunlight, a spinning satellite always presents some part of its exterior to the warming rays of the sun. The surfaces alternately heat and cool as the vehicle spins through space, providing exactly the conditions needed to produce an electrical current.

According to engineers of ITT Laboratories, Nutley, N.J., where the device was developed, the principles behind this type of energy converter hinge on the special characteristics of a family of insulating materials known as ferroelecrtics. As these ceramic substances, sandwiched between charged plates, are heated, they transfer their electrical energy to the plates, increasing the voltage. In this fashion heat is converted to electrical energy.

In a typical satellite application engineers would connect a pair of ceramic "sandwiches." Through the heating and cooling processes a current is made to flow back and forth from one to the other. A number of additional "capacitors" or ferroelectric sandwiches can be connected to increase the available voltage or current. In addition, these unusual temperature characteristics of the ceramic materials can be manipulated and tailored to meet the specific requirements of any given environment.

The converter can be used to power transmitters, receivers and similar equipment in space vehicles. It appears that extremely high voltages can be obtained to provide power for propelling space ships, ITT scientists said.

Using a single converter element the size of a dime, outputs of more than 1,000 volts already have been obtained directly without need for voltage step-up devices. By arranging elements in series, outputs of 1,000,000 volts are theoretically possible.

Engineers at ITT Laboratories have calculated that devices with a better power output per pound than existing satellite power supplies are possible.

Safer Production of Solid Propellants.—A safer and faster method for the continuous processing of solid propellants is under development by Rocketdyne, a division of North American Aviation, Inc. Popularly known as "QuickMix," the new process is expected to provide a high degree of mobility for the manufacturing of solid propellants in the field and loading into large rocket motors. Further work in applying the process to several specific propellant formulations is being performed under contract with the manufacturing methods division, A.M.C., Aeronautical Systems Center.

A "QuickMix" pilot plant, which has a minimum capacity of 500 lb. of propellant per hour, recently was loaded on a 30-ft. truck trailer and transported from California to the division's solid propellant plant at McGregor, Texas. Rocketdyne engineers have used the pilot plant to process several types of solid propellant formulations.

Principle objective of the new process is safety. In conventional methods of mixing solid propellants, the ingredients are brought together in batch mixers to produce quantities of finished propellant. As more sensitive propellants are used, and as larger quantities are processed, the hazards of fire and explosion during mixing are increased.

In the "QuickMix" process, propellant ingredients are thoroughly blended in a matter of seconds with only a very small amount of material being in the mixer at any given mo-

ment. Oxidizer is dried, sifted, and ground before being dispersed in a liquid carrier. Minor dry propellant ingredients are handled in a similar way. Liquid ingredients enter the system through a colloid mill for emulsifying with the liquid carrier. Where metallized propellants are required, the powdered metal also will be mixed with a liquid carrier, thus reducing the special hazards attendant to the handling of the fine metal powders.

The streams of oxidizer and other ingredients in their liquid carrier are blended at high speeds in a small mixer. Because of the fluidity, mixing is accomplished very rapidly with no appreciable heat build up and a very low power requirement. Propellant formation is immediate upon contact of the binder droplets and oxidizer particles.

"QuickMix" is also expected to offer advantages of low cost and versatility. A 5000-lb per hour plant would be able to operate successfully at reduced rates. For field loading of large rocket motors containing thousands of pounds of propellant, the "QuickMix" plant and raw ingredients would be transported to the proposed firing site. The mixing of "QuickMix" propellant and loading of the rocket motor in the field thus would eliminate the Herculean task of transporting huge rocket boosters from the manufacturing plant.

The system promises to be applicable to most known solid propellants and can be quickly changed over from one formulation to another.

Sunshine Powers Air Conditioners. —When Stephen Foster wrote "The sun so hot I froze to death," he may have been forecasting the truth. An experiment using the sun's heat to power air conditioning units was described at the Annual Meeting of The American Society of Mechanical Engineers in Atlantic City, in a paper presented by Melvin M. Eisenstadt, The Martin Company, Orlando, Florida, and Frank M. Flanigan and Erich A. Farber of the University of Florida. They said that using sunshine to power an air conditioning unit is a logical idea, since the hotter the sunshine, the more power it provides and the more heat a solar air conditioning system could remove. This provides peak efficiency when most needed.

The experimental unit they used works on much the same principle as a gas refrigerator, but instead of heat being supplied by a gas flame, it is supplied by the rays of the sun. The somewhat involved process by which heat can be used to cool things is called "refrigeration by absorption" and relies on a mixture of water and ammonia gas which is alternately separated and remixed. In practice, heat from the sun would be gathered by a "flat plate collector," which can be installed on the roof or near a building. Such collectors are relatively simple and inexpensive.

One problem on which the authors are working is finding the concentration of ammonia to water which would work best when the heat source is around 140 to 180 F, much lower than the temperatures of a gas flame usually used in such systems, but easily obtained with current solar heaters. They concluded that solar powered air conditioners are technically feasible, if high concentrations of ammonia, from 40 to 70 per cent, are used in the absorption system.

Alumina Powder Used for Potting. —A basically new approach in the protection of electronic components has been revealed by L. W. Kirkwood and R. S. Key of Bell Telephone Laboratories. They described how alu-

mina powder (aluminum oxide) can be used successfully to insulate, or "pot" transformers and other electrical components within hermetically sealed cans, replacing the more conventional semifluid materials such as mineral-filled asphalts which have been used in the past. A paper describing the development was presented at the National Conference on Application of Electrical Insulation in Washington, D.C. The first use of alumina in this application was made by H. S. Feder, also of Bell Laboratories.

In many applications, electronic and electrical components must have extra protection from extremely humid environments. A common practice is to seal these components hermetically inside a metal can. However, when the component is enclosed in this way, its internal temperature is increased to the point that the life of the device is seriously reduced. In conventional practice, therefore, a heat-transfer material is placed in the can to carry the heat from the device to the can, and thence to the air, in order to reduce the over-all temperature rise. Materials used for this purpose are called "potting compounds."

Popular conventional potting compounds include mineral-filled asphalts such as sand asphalt. This has good heat transfer properties, but it melts at the relatively low temperature of 135–150 C. (275–300 F.). When it melts, it expands, threatening the hermetic seal and often damaging the component.

Alumina powder does not show these disadvantages. Its melting point is over 1500 C. (2700 F.), well above the operating temperatures of any electrical apparatus. It does not expand or contract to any noticeable extent under wide fluctuations of temperature, so no strains are imposed on the component. Also, it doesn't need to be cured or vulcanized for use as some of the plastic potting compounds do, and thus no strains develop from these processes.

The powder possesses another unique advantage over asphalts, epoxy resins, and other similar potting compounds, in that it maintains its dry, granular form in use. The electrical component can be removed for inspection or repairs at will, simply by breaking the seal on the can and pouring out the powder.

Alumina is completely inert. Thus, there is no fire hazard, either during potting operations or in use, in contrast to inflammable asphalts or resins. Since alumina is stable to such high temperatures, it can be used as a single potting compound to cover the gamut of temperature ranges.

The preferred physical form of the alumina is spherical. In this shape, the granules pack well, but do not have the abrasive characteristics of more irregular shapes. Material of this type is readily available at prices competitive with conventional materials.

The test program on alumina potting compounds was performed on typical, medium-sized electronic power transformers, which were impregnated with a polymerizing varnish before potting and encasing.

Tiny Tunnel Diode.—A tiny device promising "important new solutions to electronic problems of the Space Age" has been made available to industry for experimental purposes by the Radio Corporation of America. The device, no bigger than the head of a match, is RCA's new germanium tunnel diode, and it is expected to perform important electronic functions in missile control, guidance sys-

tems, and ultra-high-speed data-processing systems. Like the electron tube and the transistor, it controls the flow of electrons that make up an electrical current.

The tunnel diode—so named because of the manner in which electrons seem to "tunnel" through the device with the speed of light—functions 100 times faster than a transistor.

Typical advantages of the tunnel diode are its ability to:

1. Expand the number of electronic functions that can be performed within a space vehicle through dramatic increases in the amount of circuitry possible in a small space.
2. Withstand cosmic and atomic radiation in vehicles traveling in outer space.
3. Greatly increase the number of "decisions" made by an electronic computer—up to a billion a second.
4. Operate on extremely low power; actually, hundreds of these RCA tunnel diodes can be operated from a one-and-one-half-volt flashlight cell.
5. Operate over an extremely wide range of temperatures—from 450 degrees below zero Fahrenheit to 212 degrees above.
6. Withstand shock and vibration —very simple, rugged construction.

In comparison to other semiconductor devices the tunnel diode possesses many outstanding advantages such as: higher frequency capability, lower noise, simpler structure, and freedom from surface effects. The latter is expected to insure extremely high reliability. The RCA tunnel diode will operate at frequencies up to a billion cycles per second.

Because of its unusual electrical properties, the tunnel diode is also extremely useful as a high-frequency oscillator, low-noise amplifier, high-speed switch, self-excited converter and clipper. All of these functions can be handled with extremely low power and low heat dissipation. The negative-resistance characteristic of the tunnel diode—the simultaneous increase in voltage with decrease in current—permits the units to supply energy rather than dissipate it through a large portion of its cycle. The instantaneous transfer of charge carriers which gives rise to this characteristic is called the tunnel effect.

RCA's initial sample types include twelve tunnel diodes designed for operation up to 1000 megacycles with power consumption ranging from 0.75 milliwatt to 3 milliwatts. Nominal peak currents (tunnel currents) range from 1.8 milliamperes to 6.8 milliamperes. For maximum usefulness of the negative resistance characteristic, the ratio of peak current to minimum current is maintained in excess of 4.5 to 1.

RCA's tunnel diode is the smallest mesa device available in the industry today. It consists of a p-n junction only 1/1000 of an inch in diameter and 80 angstroms in width (about 1/150 the wavelength of visible light). This unit is mounted in a new miniature, low inductance ceramic case specially designed for ultra-high-frequency applications. The unique package, developed at RCA's Laboratories by Dr. Charles Mueller, has an inductance of only 0.4 millimicrohenry, which is believed to be far lower than that of any available semiconductor case. This extremely low inductance is made possible by an unusual design which eliminates the use of wire leads. The entire package including ribbon connections is smaller than a match head.